

Torque

AP* Physics 1 Big Idea 3, Investigation 12

An Advanced Inquiry Lab

Introduction

The ability to make strong, rigid structures has been important ever since buildings were first constructed many thousands of years ago. In modern times, structural strength is even more important with the construction of complex bridges and skyscrapers. All these structures have the same physical property in common—they are all in static equilibrium. This laboratory activity introduces the concept of static equilibrium. Your task, as an engineer, will be to hang a “sign” over a sidewalk for the lowest material cost.

Concepts

- Torque
- Second- and third-class lever arms
- Trusses and boom supports
- Static equilibrium of a rigid body

Background

If a force, applied to an object, causes the object to rotate about a fixed point, then a net torque is said to have been applied to the object. This net torque depends on the level arm, defined as the distance from the pivot point to the point the force is applied and the amount of that force applied at a right angle to the level arm (see Figure 1).

Torque, T , is defined as the cross product of the force and the lever-arm distance, r , and is equal to the distance of the lever-arm times the magnitude of the force times the sine of the angle between the lever-arm and the force vector.

$$T = r \times F = rF\sin(\theta) \quad \text{Equation 1}$$

If the angle is anything other than 0° , then a net torque exists and rotation about the fixed point occurs. Counterclockwise rotation is designated as negative, while clockwise is labeled positive. If more than one force is acting on the lever arm, it is the sum of the torques that determines not only the extent, if any, of the rotation, but also its direction.

Static equilibrium occurs when all the forces acting on a structure are in perfect balance, that is, there is no linear or rotational movement. If a building or bridge is not in static equilibrium, the unbalanced forces, the most significant being from the force due to gravity, will eventually cause the structure to fall.

The ability to maintain static equilibrium becomes more difficult when an object must be supported from above instead of below. Long suspension bridges are generally supported by wire cables that attach to the tops of supporting bases (see Figure 2). Overhanging signs and cranes are similar to suspension bridges and rely heavily on strong cables, a supporting lever, also called a boom or truss, and cable attachments. In order to save costs, engineers attempt to limit the amount of material used to support a structure, while still maintaining a high level of strength so that the structure stays in static equilibrium for many years to come.

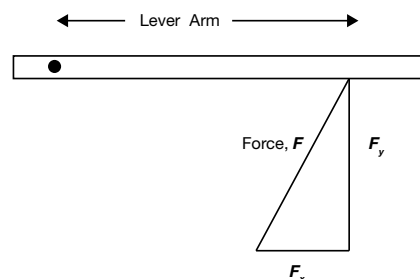


Figure 1.



Figure 2.

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A simple truss is a supporting structure consisting of a lever arm (boom) and a supporting cable. A simple truss can act as either a Class II lever or a Class III lever, depending on where the supporting cable is in relation to the supported load (see Figure 3). The fulcrum of the truss is the pivot point where it is connected to the supporting wall.

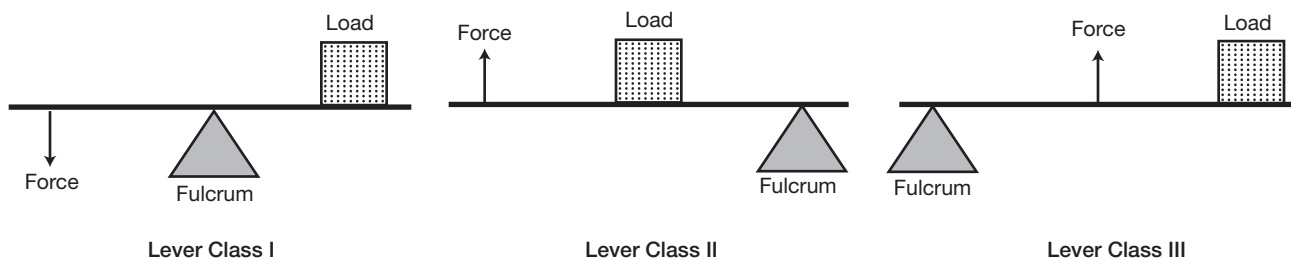


Figure 3.

The conditions for static equilibrium occur when the net force acting on the rigid body and the net torque about any point on the rigid object are both equal to zero (see Equations 2 and 3).

$$F_{\text{net}} = 0 \quad \text{Equation 2}$$

$$T_{\text{net}} = 0 \quad \text{Equation 3}$$

Experiment Overview

The lab begins with an introductory demonstration of the materials used in the guided-inquiry section of the investigation. The lever arm, the mass weight, the hangers, the spring scale and string are assembled on a support rod. The student is then instructed on taking data angles, lengths and forces. The experiment’s guided-inquiry design section presents a challenge to use the demonstration setup to model the task of hanging a “sign” over a sidewalk for the lowest material cost. The boom used to hang the sign must be attached to the wall of the building at a specific location. The sign should be hung as far away from the building as possible in order to maximize its visibility and marketing potential. Also, in order to keep the costs of the project down and still maintain building code, the “cable” material used can only support half the weight of the sign (250 g; 2.45 N), the “cable” will snap. In order to accomplish this task, experiment with the position of the sign on the boom, the angle of the boom, and the position of the supporting string.

Pre-Lab Questions

1. Use simple right-hand triangle geometry (see Figure 4) to determine the length of string (l) needed to support a 40-cm-long boom when the string is held at a 30° angle and the boom is held parallel with respect to the ground. Assume the string is attached to the wall directly above the pivot point of the boom.
2. Using your solution to *Pre-Lab Question 1* as a guide, calculate the string lengths for the different string positions and string angles. Assume the string “attachment point” is directly above the pivot point of the boom. Record these values in a data table.

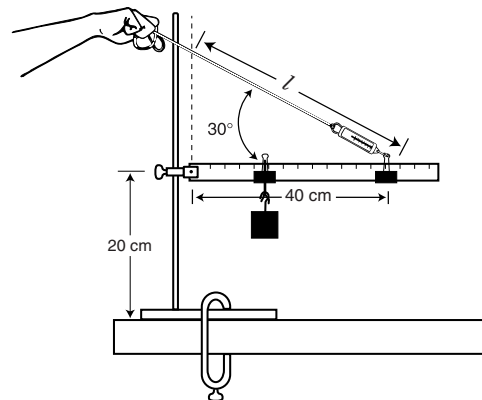


Figure 4.

Materials

- | | |
|---|----------------------------|
| Binder clips, 2 | Scissors |
| C-clamp, 3" (optional, but recommended) | Spring scale, 1000-g/10-N |
| Hooked mass, 500-g | String |
| Meter stick, half, with support stand clamp (Simple Form Truss) | Support clamp with bracket |
| Protractor | Support stand |
| Ruler | |

Safety Precautions

The materials in this lab are considered safe. Please follow normal laboratory safety guidelines.

Introductory Activity

1. Tie one end of a 50-cm string to the spring-scale handle.

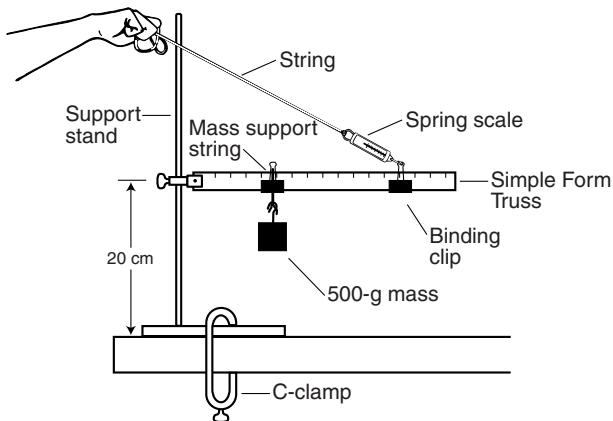


Figure 5.

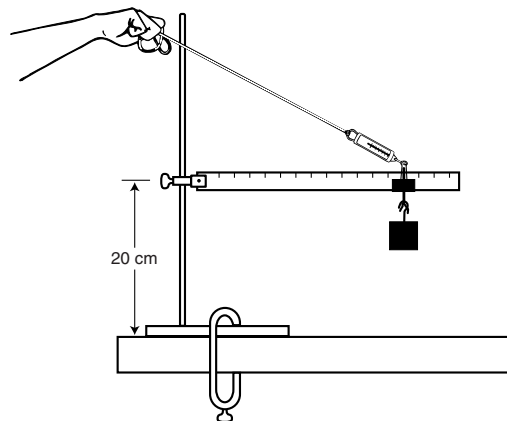


Figure 6.

2. The setup for the Simple Form Truss is shown in Figures 5 and 6.

Note 1: The scale of the spring-scale should face the measurer.

Note 2: Tie the 15-cm string around the binder clip to form a loop that is just large enough to allow the string to slide along the meter stick. (see Figure 7). The hooked mass will hang from the string and the binder clip will prevent the string from slipping. The spring-scale binder clip should be clipped on the bottom edge of the meter stick. This will prevent the clips from being pulled off the meter stick.

Note 3: (Optional) Clamp the support stand to the tabletop using a C-clamp.

Note 4: Figure 6 shows how to use the mass binder clip when the mass and string are attached at the same position on the meter stick. Only one binder clip is used for this setup.

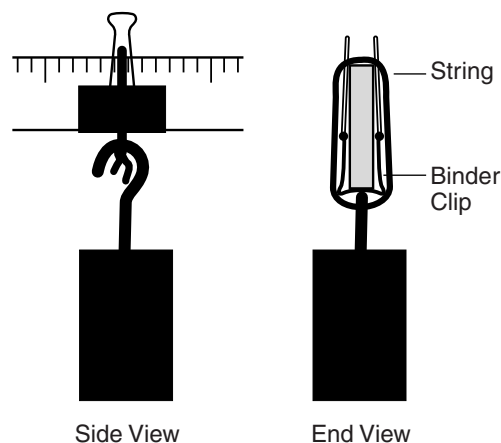


Figure 7.

Guided-Inquiry Design and Procedure

1. Obtain the Simple Form Truss, two binding clips, support stand, 500-g hooked mass, string, scissors, protractor, ruler and 1000-g spring scale.
2. Use scissors to cut two pieces of string—one approximately 50 cm and the other approximately 15 cm.

Form a working group with other students and discuss the following questions.

1. With the 500-g mass and the string attachment points fixed at given distances, does the force needed to maintain equilibrium decrease or increase when the string angle is increased? Explain.
2. If the distance from the pivot point is increased for the 500-g mass in question 1 and the string is held at a fixed angle, does the force needed to maintain equilibrium increase? Explain.
3. In the arrangement in question 1, if the string attachment point is moved closer to the pivot point and the string angle is fixed, does the force needed to maintain equilibrium increase or decrease? Explain.

- The purpose of this activity is the creation of a design for hanging a “sign” over a sidewalk for the lowest material cost. The parameters you are to work with are:
 - Sign mass: 500 g
 - Boom attachment height on wall: 20 cm
 - Maximum supporting weight of cable: 250 g (2.45 N)
 - Cost of supporting cable: \$5/cm
- Write a detailed step-by-step procedure to collect the data needed for optimizing your hanging sign model. Include all the materials and equipment that will be needed, along with all data tables.
- When all the data has been collected for a particular mass position and string angles, slowly lower the mass to the tabletop.

Disposal

Consult your instructor for appropriate storage procedures.

Opportunity for Inquiry

Use the same outline to determine the strength of a cable needed to secure a sign from the force of wind gusts, given the area of the sign and the force of the wind.

Review Questions

- Which boom setup completed the objectives in the best possible manner (i.e., the sign is hanging as far away from the wall as possible using the least amount of string)? How much did it cost for the cable?
- In order to provide the strongest support, where is the best place to hang the mass in relation to the string attachment?
- How does the angle of the string affect the tension in the string?
- Compare the force measurements for when the string and mass are attached at the same position.
- Which boom in Figure 8 would produce the most tension in the string? Why? Assume the mass in each figure is the same.

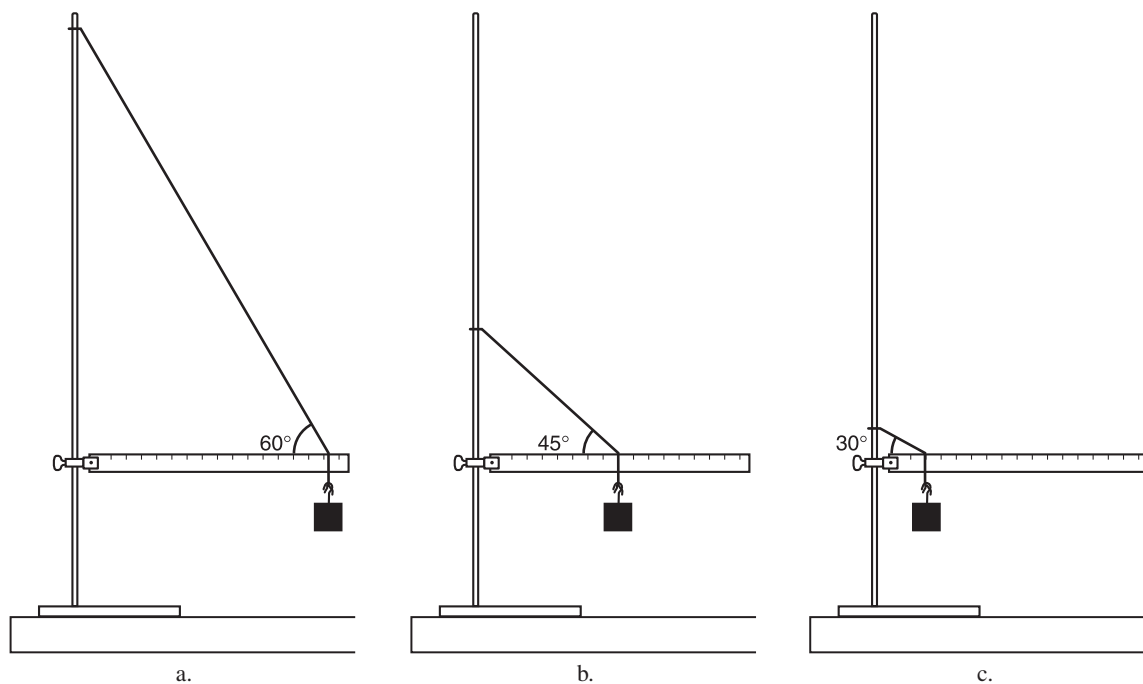


Figure 8.

AP Physics 1 Review Questions

Integrating Content, Inquiry and Reasoning

1. A series of forces are exerted on a 20-meter bar at various locations:

- A 48 N force, at 10 meters, acting at a 45° angle above the bar
- A 4 N force, at 12 meters, acting at a 90° angle above the bar
- An 88 N force, at 15 meters, acting at a 145° angle above the bar
- A 60 N force, at 8 meters, acting at a 45° angle below the bar
- A 100 N force, at 10 meters, acting at a 90° angle below the bar

Draw the vector diagram and calculate the resultant torque, if any, on the 20-meter bar.

2. A truck moves across a bridge. The truck has a mass of 1700 kg. The bridge is 1200 meters long, with 2 piers, each pier located 400 meters from the ends. At the point where the truck has travelled 700 meters across the bridge, what are the added forces on each pier? Draw the vector diagram.

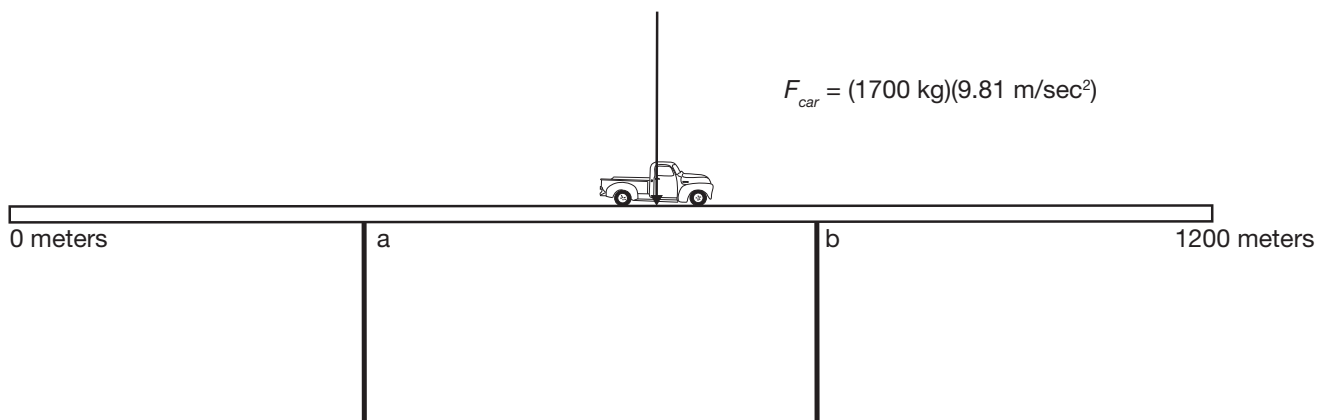


Figure 9.

3. A 40-gram mass is supported by 2 ropes. The angles θ_1 and θ_2 are 60° and 45° , respectively. Find the values for T_1 and T_2 .

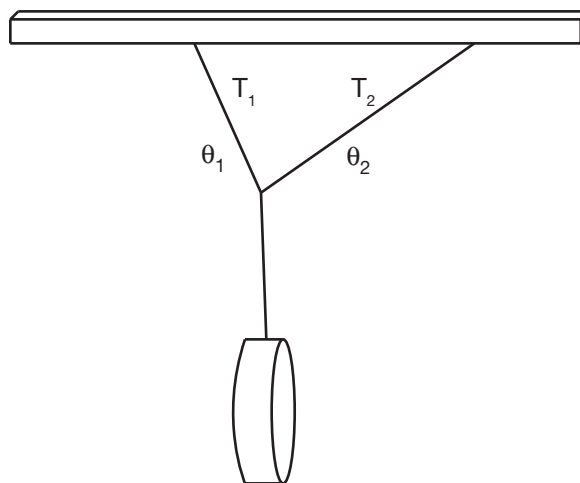


Figure 10.

Teacher's Notes

Investigation 12—Torque

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Part I. Lab Preparation

Materials Included in Kit (for 24 students working in pairs.)

- | | |
|-----------------------------------|--|
| Binder clips, 32 | String, thin, 1 ball |
| Meter sticks, half, with hole, 12 | Support clamps with brackets, nuts and bolts, 12 |

Additional Materials Required (for each group)

- | | |
|---|---------------------------|
| C-clamp, 3" (optional, but recommended) | Scissors |
| Hooked mass, 500-g | Spring scale, 1000-g/10-N |
| Protractor | Support stand |
| Meter stick | |

Time Required

This experiment can be reasonably completed in two 50-minute class periods.

Pre-Lab Preparation

Fasten the bolt through the hole in the half meter stick and through the bracket on the support stand clamp. Tighten the bolt assembly until the meter stick does not twist but can still be raised and lowered easily.

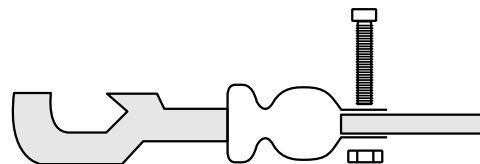


Figure 11.

Safety Precautions

The materials in this lab are considered safe. Please follow normal laboratory safety guidelines.

Disposal

All materials may be saved for future use.

Part II. Teacher Guidance

Alignment to the AP* Physics 1 Curriculum

Enduring Understandings and Essential Knowledge

A force exerted on an object can cause a torque on that object. (3F)

3F1: Only the force component perpendicular to the line connecting the axis of rotation and the point of application of the force results in a torque about that axis.

- a. The lever arm is the perpendicular distance from the axis of rotation or revolution to the line of application of the force.
- b. The magnitude of the torque is the product of the magnitude of the lever arm and the magnitude of the force.
- c. The net torque on a balanced system is zero.

Learning Objectives

3F1.1: The student is able to use representations of the relationship between force and torque.

3F1.2: The student is able to compare the torques on an object caused by various forces.

3F1.3: The student is able to estimate the torque on an object caused by various forces in comparison to other situations.

3F1.4: The student is able to design an experiment and analyze data testing a question about torques in a balanced rigid system.

3F1.5: The student is able to calculate torques on a two-dimensional system in static equilibrium, by examining a representation or model (such as a diagram or physical construction).

Science Practices

- 1.4 The student can use representations and models to analyze situations or solve problems qualitatively and quantitatively.
- 2.2 The student can justify the selection of a mathematical routine to solve problems.
- 4.1 The student can justify the selection of the kind of data needed to answer a particular scientific question.
- 4.2 The student can design a plan for collecting data to answer a particular scientific question.
- 5.1 The student can analyze data to identify patterns or relationships.
- 6.1 The student can justify claims with evidence.

Lab Hints

- Assume the mass of the boom and binding clips is negligible.
- If 500-g masses are not available, any small, heavy object can be used. A small bag of sand is one low-cost option. The design of the experiment may need to be changed slightly (having to hang the mass over the edge of a table if it is too large, etc.)
- To limit the metal fatigue in the binding clips, remove the clips from the meter stick before storing. Two additional clips have been provided if binding clips become too weak to securely clamp onto the meter stick. The binding clips can also be purchased at a local office supply store.

Teacher's Notes *continued*

Teaching Tips

- This lab is inquiry-based. Provide the students only with the necessary task and have them develop their own procedure, identifying the variables, developing a data table, and analyzing their results.
- Instruct students to slowly raise or lower the string angle, while keeping the boom parallel to the ground, until the string is at the appropriate angle according to the value in the data table. Use a protractor to measure the approximate angle of the string. See Figure 12. All angles should be measured with respect to the horizontal.
- In order to determine the length of string, use simple right-triangle geometry. Assume that the attachment point of the string is directly above the pivot point.

$$\sin\theta = \text{opposite side/hypotenuse}$$

$$\cos\theta = \text{adjacent side/hypotenuse}$$

$$\tan\theta = \text{opposite side/adjacent side}$$

Part III. Sample Data, Results and Analysis

Answers to Pre-Lab Questions

1. Use simple right-hand triangle geometry (see Figure 4) to determine the length of string (l) needed to support a 40-cm-long boom when the string is held at a 30° angle and the boom is held parallel with respect to the ground. Assume the string is attached to the wall directly above the pivot point of the boom.

$$l = (40 \text{ cm})/(\cos 30^\circ) = 46.2 \text{ cm}$$

2. Using your solution to *Pre-Lab Question 1* as a guide, calculate the string lengths for the different string positions and string angles. Assume the string "attachment point" is directly above the pivot point of the boom. Record these values in a data table.

See Sample Data Tables. Note: The string lengths were calculated assuming the boom is 50 cm long. In this setup, however, the boom is actually 49 cm long because the pivot point is at the 1-cm mark, not the zero mark. So, the value of the positions of the string and mass will be one higher than their actual distance from the pivot. Some students may recognize this condition. Whether students use the value in the data table or the actual distance, the string length will not vary considerably (by 2 at most) and will not affect the students' conclusions as long as they are consistent.

Answers to Guided-Inquiry Discussion Questions

1. With the 500-g mass and the string attachment points fixed at given distances, does the force needed to maintain equilibrium decrease or increase when the string angle is increased? Explain.

Decrease. For a given force, as the angle increases, the normal component of force also increases. To retain the same normal component of force request a smaller overall force at this increased angle.

2. If the distance from the pivot point is increased for the 500-g mass in question 1 and the string is held at a fixed angle, does the force needed to maintain equilibrium increase? Explain.

Increase. For a given force, as the distance from the pivot point increases, the amount of torque also increases. The normal component of the string force, and therefore the force on the string, must also increase to balance the increase in torque.

3. In the arrangement in question 1, if the string attachment point is moved closer to the pivot point and the string angle is fixed, does the force needed to maintain equilibrium increase or decrease? Explain.

Increase. For a given torque, as the distance from the pivot point decreases for the string, the amount of force must increase to balance the decrease in distance.

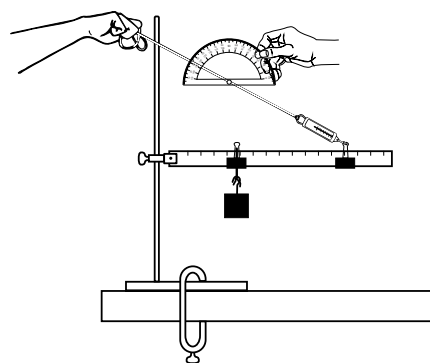


Figure 12.

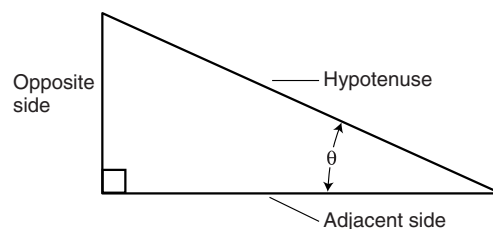


Figure 13.

Teacher's Notes *continued*

Sample Data

Test	String Clip Position	Mass Clip Position	Boom Angle	String Angle	String Length	Spring Scale Force
1	40 cm	40 cm	0°	30°	46 cm	900 g
2	40 cm	40 cm	0°	45°	57 cm	700 g
3	40 cm	40 cm	0°	60°	80 cm	600 g
4	40 cm	25 cm	0°	30°	46 cm	600 g
5	40 cm	25 cm	0°	45°	57 cm	450 g
6	40 cm	25 cm	0°	60°	80 cm	350 g
7	40 cm	10 cm	0°	30°	46 cm	275 g
8	40 cm	10 cm	0°	45°	57 cm	150 g
9	40 cm	10 cm	0°	60°	80 cm	110 g
10	25 cm	40 cm	0°	30°	29 cm	>1000 g
11	25 cm	40 cm	0°	45°	35 cm	>1000 g
12	25 cm	40 cm	0°	60°	50 cm	950 g
13	25 cm	25 cm	0°	30°	29 cm	900 g
14	25 cm	25 cm	0°	45°	35 cm	700 g
15	25 cm	25 cm	0°	60°	50 cm	600 g
16	25 cm	10 cm	0°	30°	29 cm	350 g
17	25 cm	10 cm	0°	45°	35 cm	250 g
18	25 cm	10 cm	0°	60°	50 cm	200 g
19	10 cm	40 cm	0°	30°	12 cm	>1000 g
20	10 cm	40 cm	0°	45°	14 cm	>1000 g
21	10 cm	40 cm	0°	60°	20 cm	>1000 g
22	10 cm	25 cm	0°	30°	12 cm	>1000 g
23	10 cm	25 cm	0°	45°	14 cm	>1000 g
24	10 cm	25 cm	0°	60°	20 cm	>1000 g
25	10 cm	10 cm	0°	30°	12 cm	850 g
26	10 cm	10 cm	0°	45°	14 cm	700 g
27	10 cm	10 cm	0°	60°	20 cm	600 g

Teacher's Notes *continued*

Answers to Review Questions

1. Which boom setup completed the objectives in the best possible manner (i.e., the sign is hanging as far away from the wall as possible using the least amount of string)? How much did it cost for the cable?

From the experimental results, the best setup is Test 17 in which the mass is positioned at 10 cm, and the string attachment is at 25 cm. This produced a string tension of 250 g. The position of the mass ("sign") is at 10 cm from the ring stand (wall). The amount of string that was needed was 35 cm, resulting in a material cost of \$165. This is the least expensive design.

2. In order to provide the strongest support, where is the best place to hang the mass in relation to the string attachment?

The best place to position the mass in relation to the string is to place the mass as close to the wall as possible and attach the string as far from the wall as possible.

3. How does the angle of the string affect the tension in the string?

As the angle of the string increases, the tension in the string decreases (all other variables being equal).

4. Compare the force measurements for when the string and mass are attached at the same position.

The force measurements for when the string and mass are attached at the same position are very similar. It does not matter where on the boom the string and mass are attached, so long as they are attached at the same point. Only the angle of the string affected the tension in the string.

5. Which boom in Figure 8 would produce the most tension in the string? Why? Assume the mass in each figure is the same.

The boom shown in Figure 8c would result in the highest string tension. This is because the angle of the string is the only important variable when the mass and string are attached at the same location. The smaller the angle of the string leads to a larger tension in the string. Therefore, Figure 8c would show the highest tension in the string.

Answers to AP Physics 1 Review Questions

1. A series of forces are exerted on a 20-meter bar at various locations:

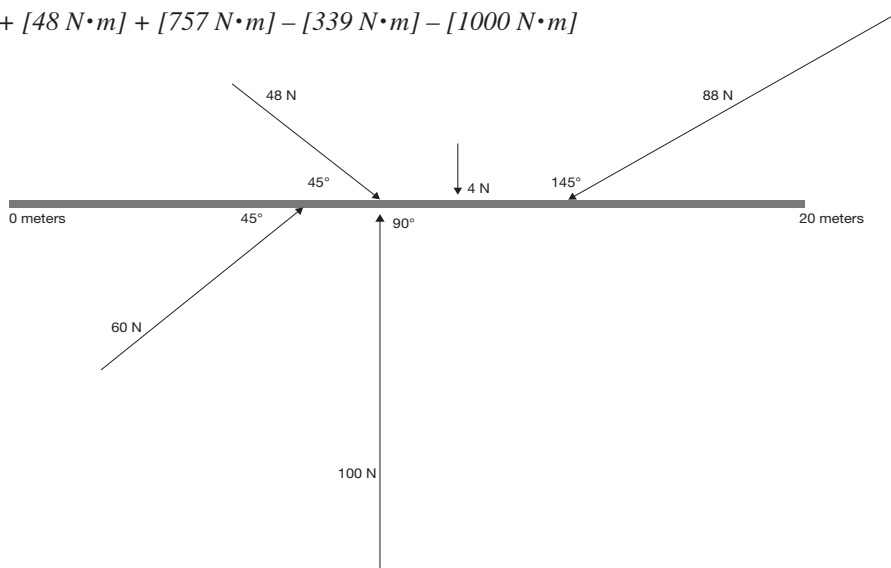
- A 48 N force, at 10 meters, acting at a 45° angle above the bar
- A 4 N force, at 12 meters, acting at a 90° angle above the bar
- An 88 N force, at 15 meters, acting at a 145° angle above the bar
- A 60 N force, at 8 meters, acting at a 45° angle below the bar
- A 100 N force, at 10 meters, acting at a 90° angle below the bar

Draw the vector diagram and calculate the resultant torque, if any, on the 20-meter bar.

$$\Sigma = [48 \text{ N} \sin(45^\circ) \times 10 \text{ m}] + [4 \text{ N} \times 12 \text{ m}] + [88 \text{ N} \times \sin(35^\circ) \times 15 \text{ m}] - [60 \text{ N} \times \sin(45^\circ) \times 8 \text{ m}] - [100 \text{ N} \times 10 \text{ m}]$$

$$\Sigma = [339 \text{ N}\cdot\text{m}] + [48 \text{ N}\cdot\text{m}] + [757 \text{ N}\cdot\text{m}] - [339 \text{ N}\cdot\text{m}] - [1000 \text{ N}\cdot\text{m}]$$

$$\Sigma = -195 \text{ N}$$

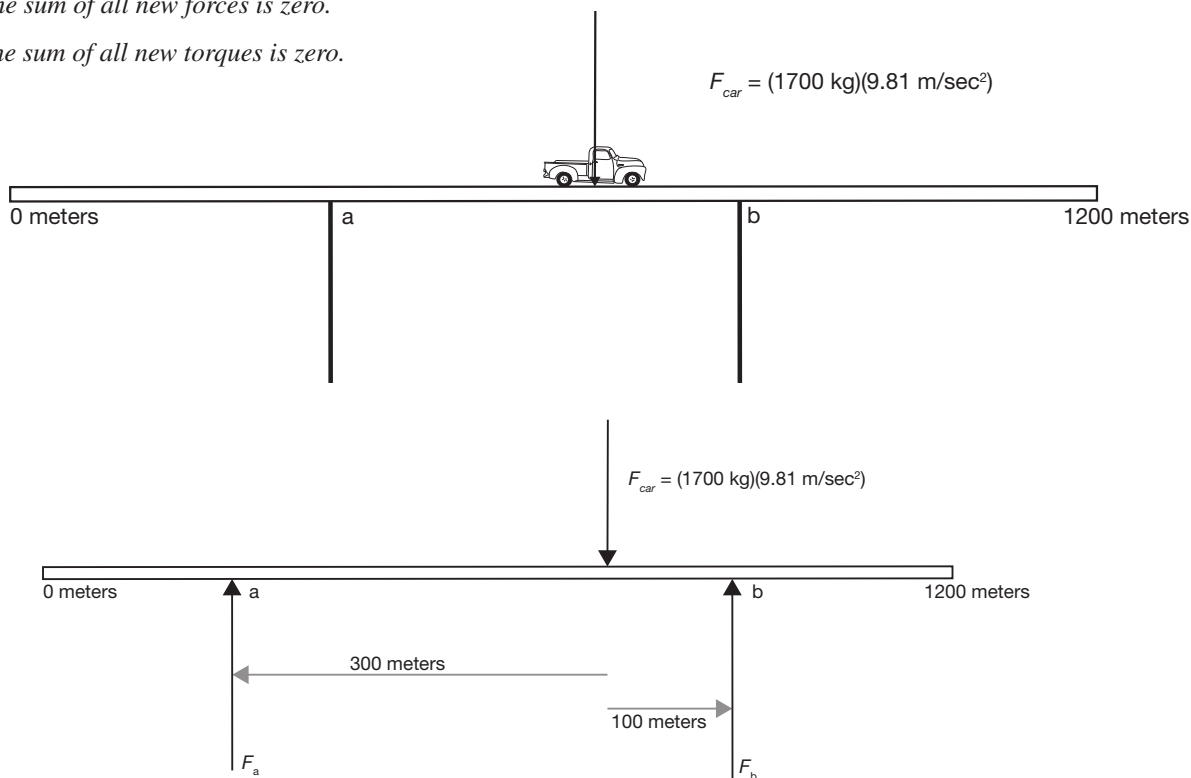


Teacher's Notes *continued*

2. A car moves across a bridge. The car has a mass of 1700 kg. The bridge is 1700 meters long, with 2 piers, each pier located 400 meters from the separate ends. At the point where the car has travelled 700 meters across the bridge, what are the added forces on each pier? Draw the vector diagram.

Since all parts of the bridge are in static equilibrium, two conditions apply:

- The sum of all new forces is zero.
- The sum of all new torques is zero.



For condition a: $F_a + F_b - F_{car} = 0$

For condition b: $T_a = T_b = T_{car} = 0$

$$F_{car} = (1700)(9.81) \text{ N} = 16677 \text{ N}$$

$$16677 \text{ N} = F_a + F_b$$

3. A 40-gram mass is supported by 2 ropes. The angles θ_1 and θ_2 are 60° and 45° , respectively. Find the values for T_1 and T_2 .

At point a, $\Sigma F = 0$

$$T_m + (-T_1) + (-T_2) = 0$$

$$T_m = T_1 + T_2 \text{ and } T_m = mg$$

The torque can also be calculated at points a and b:

$$T_a = (300 \text{ m})(16677 \text{ N}) - (400 \text{ m})(F_b) = 0, \text{ rearranging:}$$

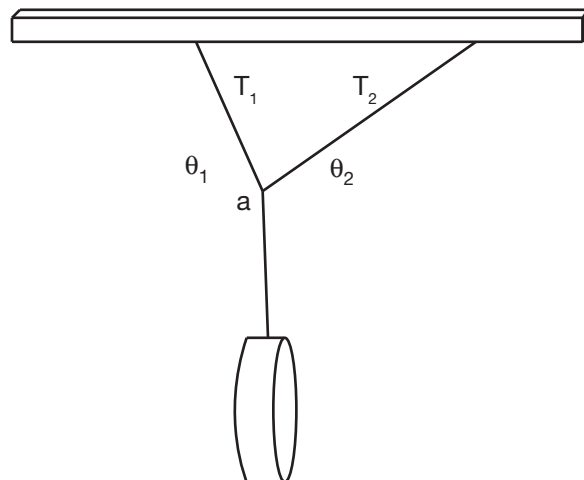
$$(300 \text{ m})(16677 \text{ N}) = (400 \text{ m})(F_b)$$

$$(0.75)(16677 \text{ N}) = F_b = 12507 \text{ N}$$

$$T_b = -(100 \text{ m})(16677 \text{ N}) + (400 \text{ m})(F_a) = 0$$

$$(100 \text{ m})(16677 \text{ N}) = (400 \text{ m})(F_a)$$

$$(0.25)(16677 \text{ N}) = F_a = 4170 \text{ N}$$



Teacher's Notes *continued*

Breaking these forces into their x and y components:

$$T_m = T_{mx} + T_{my}; \text{ since } T_{mx} = 0, T_m = T_{my}$$

$$T_1 = T_{1x} + T_{1y}$$

$$T_2 = T_{2x} + T_{2y}$$

$$\text{Therefore, } T_{my} = T_{1x} + T_{1y} + T_{2x} + T_{2y}$$

The y components are additive, as are the x components:

$$T_{my} = T_{1y} + T_{2y}$$

$$T_{mx} = 0 = -T_{1x} + T_{2x} \text{ or } T_{1x} = T_{2x}$$

Using trigonometry:

$$T_{1x} = (\cos 60^\circ)T_1$$

$$T_{2x} = (\cos 45^\circ)T_2$$

$$T_1 = [(\cos 45^\circ)/(\cos 60^\circ)]T_2 = 1.4 T_2$$

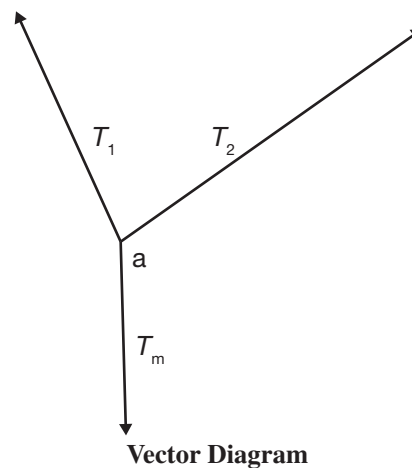
$$T_{my} = T_{1y} + T_{2y} = (\sin 60^\circ)T_1 + (\sin 45^\circ)T_2 = (\sin 60^\circ)1.4T_2 + (\sin 45^\circ)T_2$$

$$T_{my} = [(\sin 60^\circ)1.4 + (\sin 45^\circ)]T_2$$

$$T_{my} = (1.21 + 0.707)T_2 = 1.92 T_2$$

$$T_2 = T_{my}/1.92 = mg/1.92 = (0.040 \text{ kg})(9.81 \text{ m/sec}^2)/1.92 = 0.203 \text{ N}$$

$$T_1 = 1.4 T_2 = 0.287 \text{ N}$$



Torque—Advanced Inquiry Laboratory Kit and supporting supplies are available from Flinn Scientific, Inc.

Catalog No.	Description
AP7733	Torque—Advanced Inquiry Laboratory Kit
AP4550	Support Stand, Economy Choice
OB2024	Hooked Weight, 500-g
OB2066	Hook Weight Set, Economy Choice
AP4665	Spring Scale, 1000 g/10-N
AP9286	Protractor, 180°

Consult your *Flinn Scientific Catalog/Reference Manual* for current prices.

Electromagnetic Induction

AP* Physics 2 Big Idea 4, Investigation 10

An Advanced Inquiry Lab

Introduction

Electricity and magnetism are just two parts of the unifying concept of electromagnetism. Once it was known that electric currents produce magnetic fields, it was natural to wonder if magnetic fields could produce electric currents. Devices such as AC generators, microphones and MRI machines all became possible due to an understanding of electromagnetic induction.

Concepts

- Electromagnetic induction
- Faraday's law
- Magnetic flux
- Lenz's law

Background

A magnetic field is produced by moving charged particle, such as an electron. A current-carrying wire, in which many moving charged particles are the source of current, produces a magnetic field that surrounds the wire. When current travels around a wire loop, the magnetic field produced by the current is strongest at the center of the loop. The direction of the magnetic field produced by a looping current is perpendicular to the plane of the loop (see Figures 1 and 2).

When many current-carrying loops are grouped together in a line, such as with a coil of wire, the current travels in the same direction in all the loops and the ensuing superposition of the magnetic field produced by each loop provides a stronger net magnetic field. This phenomenon of an induced magnetic field was discovered by Hans Christian Oersted in 1819 when he noticed the deflection of a compass needle when a circuit was connected near the compass. This was the first observation of the connection between electricity and magnetism, whose combined interactions are known as *electromagnetism*.

In 1831, Michael Faraday (1791–1867) discovered through careful experimentation that "any change in the magnetic environment of a coil of wire will cause a voltage or *emf* to be 'induced' in the coil." The relationship states that the average magnitude of the induced emf , ϵ_{in} , in a coil with N loops is the magnitude of the ratio of the magnetic flux change through the loop $\Delta\Phi$ to the time interval Δt during which that flux change occurred, multiplied by the number of loops. This is known as *Faraday's law*. The number of magnetic field lines through a two-dimensional area, most commonly the area of a coil, is known as *magnetic flux*. The induced emf in a coil due to changing magnetic flux is given by

$$\text{emf} = N \left| \frac{\Delta\Phi}{\Delta t} \right| \quad \text{Equation 1}$$

where

emf = electromotive force or voltage (V),

N = number of turns in a coil of wire,

$\Delta\Phi$ = change in magnetic flux (weber or Wb),

Δt = time interval.

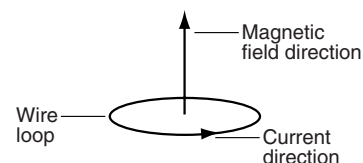


Figure 1.

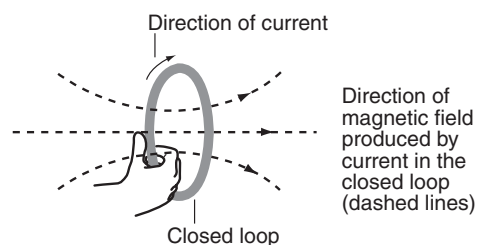


Figure 2.

Magnetic flux is given by

$$\Phi = BA\cos\theta$$

Equation 2

where

Φ = magnetic flux,

B = magnitude of magnetic field (T, tesla),

A = area the field lines pass through (m^2),

θ = angle between the magnetic field lines and a vector perpendicular to the area.

Magnetic flux can change if the magnetic field or area changes. If the field and area are held constant, magnetic flux can still change if the angle between the perpendicular area and magnetic field lines changes.

Henrich Lenz (1804–1865) further investigated Faraday's law and discovered that the induced current in a coil travels in a direction that aligns with the principle of conservation of energy—the induced magnetic field **opposes** the change in magnetic flux through the coil's area. If the increasing magnetic flux through a loop led to an induced magnetic field in the **same** direction as the external field, then the magnetic flux would continue to increase, leading to a greater induced current and so on. Energy would not be conserved and the wire would melt. Instead, if the magnetic flux through a coil is increasing, the direction of the induced current's magnetic field would lead to a decrease in the flux. If the magnetic flux through a coil is decreasing, then the direction of the induced current's magnetic field would lead to an increase in flux. This is known as Lenz's law.

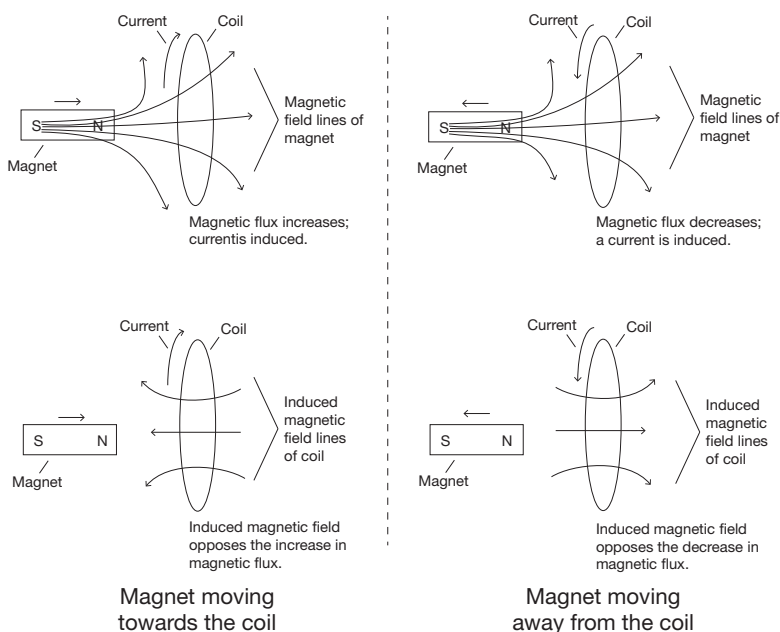


Figure 3.

Experiment Overview

The purpose of this advanced inquiry investigation is to understand the unity of electricity and magnetism through a discovery of electromagnetic induction. An introductory activity uses a coil of wire, galvanometer and magnet to investigate how magnetic flux induces a current on a coil of wire. The guided-inquiry activity is a challenge to design a mechanism using available materials that will light an LED through an AC generator.

Pre-Lab Questions

1. What is magnetic flux?
2. A magnetic field of 2 T is perpendicular to a 5 cm \times 5 cm square coil. What is the magnitude of magnetic flux through the coil?
3. What are the units for magnetic flux? *Hint:* Refer to Equation 2.
4. Draw how the magnetic field would look around a bar magnet.

Materials

Cardboard tube	Magnet wire, 30 gauge
Connector cords with alligator clips, 2	Neodymium magnets, 2
Galvanometer, -500 to $+500$ μA	Plastic jar, 60 mL
Iron nail	Sandpaper sheet
LED, red	Scissors
Magnet wire, 1 m	Transparent tape
Magnet wire, 4 m	

Safety Precautions

The materials in this lab are considered nonhazardous. Care should be taken when wrapping and unwrapping the wire. The pointed ends of the wire are dangerous to eyes. Neodymium magnets are very strong and will accelerate towards each other and other metal objects very quickly. Care should be taken to avoid unexpected and significant pinches of skin. Wear safety glasses. Please follow all normal laboratory safety guidelines.

Introductory Activity

1. Obtain 4-m and 1-m lengths of magnet wire, a 60-mL plastic jar, a piece of sandpaper, two neodymium magnets, two connector cords, some transparent tape and a galvanometer.
2. Use sandpaper to completely sand off about 2 cm of red enamel at both ends of each piece of magnet wire to expose the shiny copper underneath.
3. Leaving about 3 cm of magnet wire free to be connected with the alligator clips later, use a small amount of transparent tape to secure one end of one of the 4-m magnet wire piece to the outside of the 60-mL plastic jar. See Figure 4.
4. Remove the lid from the jar and begin wrapping the wire just below the lip of the jar so the wire stays in place. Tightly and neatly wrap the wire around the jar as shown in Figure 4.
5. Leave about 3 cm of magnet wire at the end and use a small amount of tape to secure the loose end of the magnet wire to the jar. Make sure the free ends of the magnet wire are accessible and can be connected to the alligator clips.
6. Attach alligator clips to the free ends of the magnet wire and connect the cords to the inputs on the galvanometer.
7. Obtain the neodymium magnets. They should be “stuck” together.
8. Rapidly move the magnet toward the coil and observe the galvanometer needle. Repeat the process while moving the magnet slowly. Record observations of the galvanometer needle.
9. Hold the magnet stationary inside the coil and observe the galvanometer needle. Record any observations.
10. From inside the coil, pull the magnet away from the coil and observe the galvanometer needle. Compare the direction of the deflection to what was observed in step 8. Record your observations.
11. Move the magnet to and from the coil, varying the speed. See Figure 5. Record your observations.
12. Rotate the magnet 90° . Observe if the galvanometer registers a current during the rotation. *Note:* Keep the magnet the same distance away from the coil while rotating.
13. With the magnet now oriented so its poles are parallel to the plane of the coil (see Figure 6), oscillate the magnet back and forth and record your observations.
14. Remove the coil from the jar.
15. Repeat steps 3–6 for the 1-m length of magnet wire.
16. Repeat steps 8–13 for the 1-m length of coil. Record your observations and compare the “magnitude” of the deflection of the galvanometer needle.

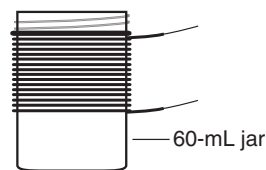


Figure 4.

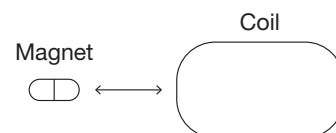


Figure 5.

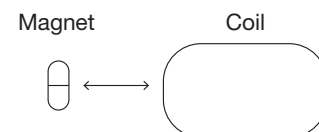


Figure 6.

Analyze the Results:

- Does the galvanometer needle deflect more when the magnet approaches the coil rapidly or slowly?
- How does the orientation of the magnet affect the observed current?
- Does the needle deflect when the magnet is at rest?
- Does rotating the magnet induce a current in the coil?
- When the magnet is oscillated to and from the coil, the needle moves from side to side indicating a current that changes direction. In your own words, why does the current change direction? Consider Lenz's law in your explanation.

Guided-Inquiry Design and Procedure

Form a working group with other students to discuss the following questions.

1. What variables affect the value of an induced emf in a coil of wire?
2. If a magnet is spinning inside of a coil of wire, how would an increasing rate of spin relate to the induced current?
3. Using the following materials, design a device that can turn mechanical energy into electrical energy and light an LED: cardboard tube, iron nail, LED, 30 gauge magnet wire, two neodymium magnets, sandpaper, transparent tape, ruler and scissors.
4. Before carrying out the desired procedure, submit your design plan to your instructor for approval.

Analyze the Results:

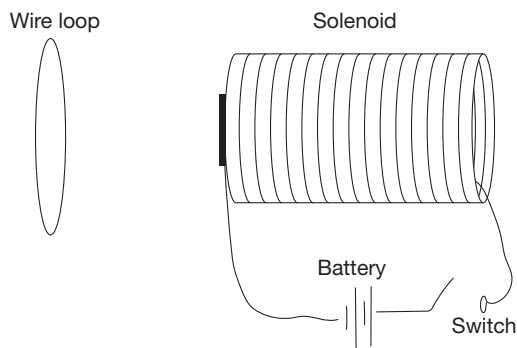
- In your own words, write a detailed summary of any errors and challenges you encountered. Include a list of variables that could affect the emf generated and explain what variable(s) you focused on in your experimental design and why.

Opportunities for Inquiry

With the use of the galvanometer or other means of detecting current, design a procedure to quantify the effect of each variable on the induced current.

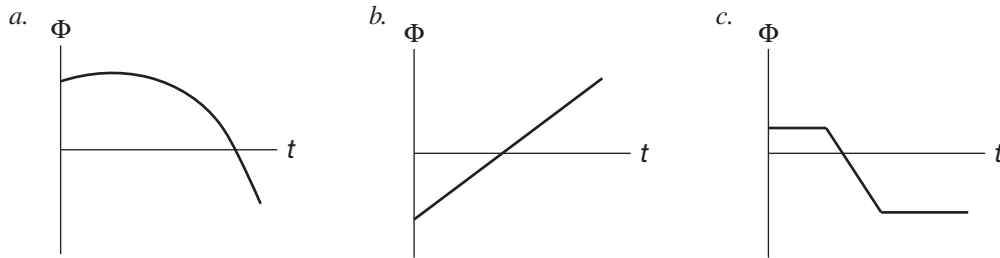
AP Physics 2 Review Questions***Integrating Content, Inquiry and Reasoning***

1. Recall that a current-carrying wire produces a magnetic field. The magnetic field of a long line of current-carrying wire loops is analogous to the field of a bar magnet. The diagram below shows a copper wire loop near a *solenoid*, a long line of wire loop. The switch in the circuit is initially open.

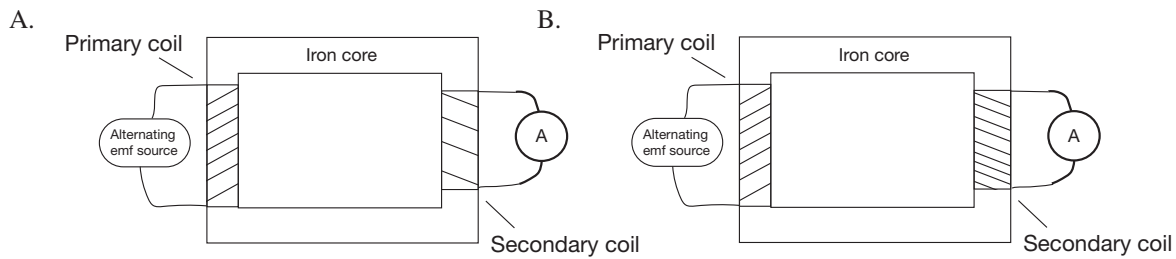


- a. Predict whether current will flow through the wire of the loop in each of the following cases. Explain your reasoning.
- Just after the switch has been closed.
 - A long time after the switch has been closed.
 - Just after the switch has been reopened.
 - A long time after the switch has been reopened.
2. An induction stovetop has a smooth surface. When on high, the surface does not feel warm, yet ramen noodles are quickly cooked when cooked in a metal bowl. However, when an attempt is made to cook the noodles in a ceramic bowl, the noodles remain cold. Explain how the stovetop works.

3. The magnetic flux through three different coils is changing as shown in the figures below. For each situation, draw a corresponding graph showing qualitatively how the induced emf changes with time.



4. A transformer consists of two coils, each wrapped around an iron core. The core confines the magnetic field produced by the electric current in one coil so that it passes through the second coil without spreading outside. The primary coil in each situation below is connected to identical emf sources.



Which ammeter would register the higher secondary emf output? Explain your reasoning.

Teacher's Notes

Investigation 10—Electromagnetic Induction

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Part I. Lab Preparation

Materials Included in Kit (for 24 students working in groups of three)

Cardboard tubes, 8	Magnet wire, 22 gauge, 1
Connector cords with alligator clips, 16	Magnet wire, 30 gauge, 8
Galvanometer, 1	Neodymium magnets, 16
Iron nails, 8	Plastic jars, 60 mL, 8
LEDs, red, 16	Sandpaper sheet, 9" × 11"

Additional Materials Required (for each lab group)

Galvanometer	Transparent tape
Scissors	

Time Required

This laboratory activity can be completed in two 50-minute class periods. It is important to allow time between the *Introductory Activity* and the *Guided-Inquiry Design and Procedure* for students to discuss and design the guided-inquiry procedures. Also, all student-designed procedures must be approved for safety before students are allowed to implement them in the lab. *Pre-Lab Questions* may be completed before lab begins the first day.

Pre-Lab Preparation

1. Measure and cut eight 4-m lengths of 22-gauge magnet wire and eight 1-m lengths of 30-gauge magnet wire; each group receives one of each.
2. Cut eight 1" × 4" pieces from the sandpaper sheet.

Safety Precautions

The materials in this lab are considered nonhazardous. Care should be taken when wrapping and unwrapping the wire. The pointed ends of the wire are dangerous to eyes. Neodymium magnets are very strong and will accelerate towards each other and other metal objects very quickly. Care should be taken to avoid unexpected and significant pinches of skin. Remind students to wear safety glasses. Please follow all normal laboratory safety guidelines.

Teacher's Notes *continued*

Disposal

All materials may be saved and stored for future use. Do not store magnets near electronics.

Part II. Teacher Guidance

Alignment to AP* Physics 2 Curriculum Framework

Enduring Understandings and Essential Knowledge

The electric and magnetic properties of a system can change in response to the presence of, or changes in, other objects or systems. (4E)

4E2: Changing magnet flux induces an electric field that can establish an induced emf in a system.

- Changing magnetic flux induces an emf in a system, with the magnitude of the induced emf equal to the rate of change in magnetic flux.
- When the area of the surface being considered is constant, the induced emf is the area multiplied by the rate of change in the component if the magnetic field is perpendicular to the surface.
- When the magnetic field is constant, the induced emf is the magnetic field multiplied by the rate of change in area perpendicular to the magnetic field.
- The conservation of energy determines the direction of the induced emf relative to the change in the magnetic flux.

Learning Objectives

4E2.1: The student is able to construct an explanation of the function of a simple electromagnetic device in which an induced emf is produced by a changing magnetic flux through an area defined by a current loop (i.e., a simple microphone or generator) or of the effect on behavior of a device in which an induced emf is produced by a constant magnetic field through a changing area. [See Science Practice 6.4]

Science Practices

- The student can describe representations and models of natural or man-made phenomena and systems in the domain.
- The student can collect data to answer a particular scientific question.
- The student can make claims and predictions about natural phenomena based on scientific theories and models.

Lab Hints

- When building the AC generator in the *Guided-Inquiry Design and Procedure*, it may be beneficial to test the coil using a multimeter's "continuity beeper" function to ensure you have a complete circuit. If the test indicates you do not have a complete circuit, remove more enamel from the ends of the wire. Check the resistance to ensure the total resistance is less than $100\ \Omega$.
- The magnet wire can be reused for additional classes as long as it is unwrapped carefully so that there are no kinks in the wire. When removing the magnet wire from the round form (plastic containers), pull the magnet wire firmly and allow the round object to spin between your fingers as the wire unwraps. This will typically produce relatively straight wire with no kinks. Holding the round object secure while unwrapping the wire generally results in curled wire and can easily produce kinks.
- To facilitate winding the coils, spools can be set up on a stand or stabilized rod so it can be held in place but rotate freely.
- The magnets are fragile and may crack or chip if they are dropped or allowed to snap together with excessive force. Use caution when handling.
- In order to preserve the coil in the instance that the wire is cut too short to light the LED (due to a lack of coils), sand the ends of the coiled wire and the wire spool, and twist them together. Test on the edges of the shaved region to make sure you have a circuit through the twisted wires, and continue spooling.
- Dimming the lights in the room may make it easier to observe the flickering LED when the magnets are being spun.
- Taping the magnets to a stirring rod or long pencil may help in handling the magnet for making observations in the *Introductory Activity*.

Teacher's Notes *continued*

Teaching Tips

- The *Introductory Activity* is a qualitative exploration of the effect of changing magnetic flux on a coil's area. The investigation and subsequent observations do not require a quantitative approach and students should focus on the orientation of the magnet and the specific movements that register a needle deflection on the galvanometer. The activity provides a hands-on introduction to the phenomenon of induced current in a coil due to changing the magnetic flux relative to a coil's area.
- If available, a compass can be used to designate the poles of the magnets used in the *Introductory Activity* so students know the orientation of the field lines due to the magnets when exploring induced current with the galvanometer.
- A complete circuit is essential for success in the *Guided-Inquiry Activity*. If a voltage drop over the wire is not produced, the LED will not light. Verify that the ends of the wires are well sanded and not touching each other to ensure good conductivity.
- Multimeters can be used to quantitatively test the amount of AC current and voltage generated by the spinning magnets in the coil.
- Upon completing the *Introductory Activity* students should be able to design a procedure in the *Guided-Inquiry Design and Procedure* where the spinning of the magnets inside the coil are the source of emf for the LED. It is important to survey each group and ask probing questions to guide them to this discovery.

Part III. Sample Data, Results and Analysis

Answers to Pre-Lab Questions

1. What is magnetic flux?

Magnetic flux is defined as the number of magnetic field lines through an area, such as the area of a coil.

2. A magnetic field of 2 T is perpendicular to a 5 cm × 5 cm square coil. What is the magnitude of magnetic flux through the coil?

$$\Phi = BA\cos\theta$$

$$B = 2 \text{ T}$$

$$A = 5 \text{ cm} \times 5 \text{ cm} = 0.05 \text{ m} \times 0.05 = 0.0025 \text{ m}^2$$

$$\theta = 0^\circ \text{ because the field is perpendicular to the coil}$$

$$\Phi = 2 \times 0.0025 \times \cos(0)$$

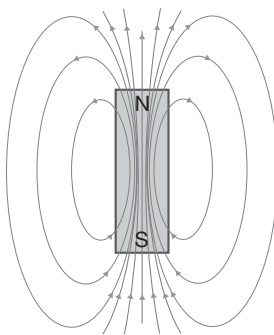
$$\Phi = 2 \times 0.0025 \times 1$$

$$\Phi = 0.005 \text{ Wb}$$

3. What are the units for magnetic flux? *Hint: Refer to Equation 2.*

Since the units for a magnetic field B is T, the tesla, and the units for area is m^2 , then multiplying the two quantities results in $\text{T} \times \text{m}^2$, the units for magnetic flux.

4. Draw the magnetic field around a bar magnet.



Teacher's Notes *continued*

Sample Data and Discussion for Introductory Activity

Magnet movement	Galvanometer needle observation
Motionless	No needle deflection
Moving towards coil quickly	A needle deflection to the right
Moving towards coil slowly	Smaller needle deflection to the right
Moving away from coil quickly	Needle deflection to the left
Moving away from coil slowly	Smaller needle deflection to the left
Magnet moved back and forth in front of coil	Needle oscillates right and left. It is deflected further with faster movement.
Oriented so the poles do not point into the coil and moved towards the coil	No deflection seen
Rotated in front of coil	Needle oscillates right and left. It is deflected further the faster the rotation

Analyze the Results:

- Does the galvanometer needle deflect more when the magnet approaches the coil rapidly or slowly?

The needle deflects more when the coil is approached rapidly by the magnet.

- How does the orientation of the magnet affect the observed current?

When the poles of the magnet are parallel to the plane of the coil, no induced current is observed when approaching the magnet. However, when the poles of the magnet are perpendicular to the plane of the coil, an induced current is clearly observed.

- Does the needle deflect when the magnet is at rest?

No, the needle does not deflect when the magnet is at rest, regardless of the orientation.

- Does rotating the magnet induce a current in the coil?

Yes, when the magnet is rotated, a current is induced in the coil.

- When the magnet is oscillated to and from the coil, the needle moves from side to side indicating a current that changes direction. In your own words, why does the current change direction? Consider Lenz's law in your explanation.

The current changes direction depending on whether the external magnetic flux is increasing or decreasing. According to Lenz's law, the induced current in the coil opposes the change in magnetic flux through the coil's area. If the flux is increasing, the induced current will generate a magnetic field that opposes the direction of the external magnetic field that is the cause of the changing flux. When the magnet is pulled away from the coil, the current reverses and generates a magnetic field in the direction of the decreasing external magnetic field. This is a consequence of the conservation of energy.

Sample Answers and Discussion for Guided-Inquiry Activity

Discussion Questions

1. What variables affect the value of an induced emf in a coil of wire?

The variables that affect the value of an induced emf in a coil of wire are the number of wire loops in the coil, the rate of change in the strength of the magnetic field, the rate of change of the coil's area, the rate of change of the angle (angular velocity of a spinning magnet or coil) that the magnetic field lines make with the plane of the loop.

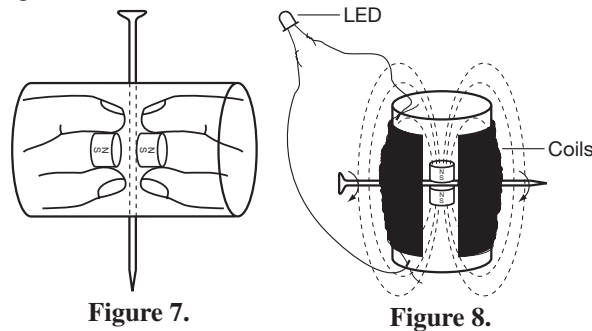
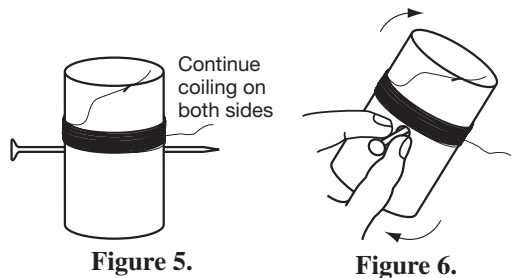
2. If a magnet is spinning inside a coil of wire, how would an increasing rate of spin relate to the induced current?

An increasing rate of spin means an increase in the rate of change of the angle that the magnetic field lines make with the plane of the loop. This means an increase in the rate of change in magnetic flux. An increase in the rate of change of magnetic flux directly increases the magnitude of the induced current.

Teacher's Notes *continued*

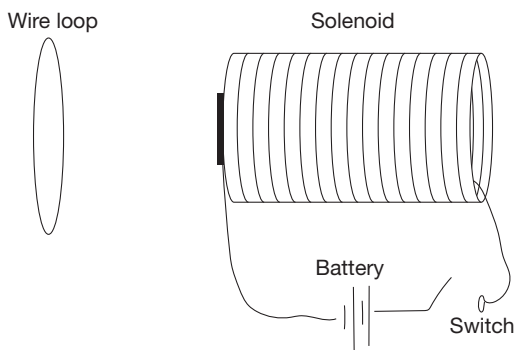
Sample Design Solution

1. Measure the length of the cardboard tube and mark the center.
2. Push the tip of the nail through the cardboard tube at the center mark and drop the nail straight down. Poke a hole through the other side as well.
3. Make a short (0.5 cm) diagonal cut at the top of the tube to hold the wire.
4. Unwind a small length from the coil of magnet wire (about 10 cm), and hook it into the cut.
5. Wind the wire around the cardboard tube at least 300 times. See Figure 5. Be cautious to not pull too tightly, as the thin wire may snap. *Note:* The more coils, the easier it will be to light the LED.
6. Once you have the desired length, spool a bit of excess and cut the wire. *Note:* This step would be skipped if the student intends to vary the number of turns.
7. Make another diagonal cut on the opposite end of the cardboard-tube generator and secure the wire end in it.
8. Use the sandpaper to remove the enameled coating on the two ends of the wire coiled around the generator.
9. Widen each hole by inserting the nail through one hole at a time and then wiggle the nail using circular motions until the nail can spin easily.
10. Test the nail's ability to spin by holding the nail steady, then spinning the cardboard tube. See Figure 6. The tube should spin easily, without much friction to slow it down. If not, repeat step 10. Leave the nail in its slots.
11. Carefully detach the magnets from each other. Slowly bring one inside the generator, gripping it loosely so it will attach to the nail on its own. The magnet should attach by a flat circular end (one of its poles).
12. Reach in with your fingers and grip the magnet attached to the nail to hold it in place. Grasp the other magnet firmly, and slowly and carefully insert it into the other end of the tube. If opposite poles are facing each other, a pulling force will be felt. If a force of repulsion is felt, flip the magnet around. Gripping both magnets firmly, bring the second magnet as close as possible before releasing it, allowing it to attach to the other side of the nail. See Figure 7.
13. Wind each shaved end of the wire around the LED's leads, ensuring the leads do not touch each other.
14. Turn off or dim the lights in the room and spin the nail with both hands. Increase the spinning speed until you can see the LED flicker on and off. See Figure 8.



Answers to AP Physics 2 Review Questions

1. Recall that a current-carrying wire produces a magnetic field. The magnetic field of a long line of current-carrying wire loops is analogous to the field of a bar magnet. The diagram below shows a copper wire loop near a *solenoid*, a long line of wire loop. The switch in the circuit is initially open.



- a. Predict whether current will flow through the wire of the loop in each of the following cases. Explain your reasoning.
 - i. Just after the switch has been closed.

Yes, current will flow in the wire because there will be an increase in magnetic flux through the area of the loop.

Teacher's Notes *continued*

ii. A long time after the switch has been closed.

No, there will be no current flow because there is no changing magnetic flux to induce a current.

iii. Just after the switch has been reopened.

Yes, current will flow in the wire due to a decrease in magnetic flux but will flow in the opposite direction as to when the switch was closed.

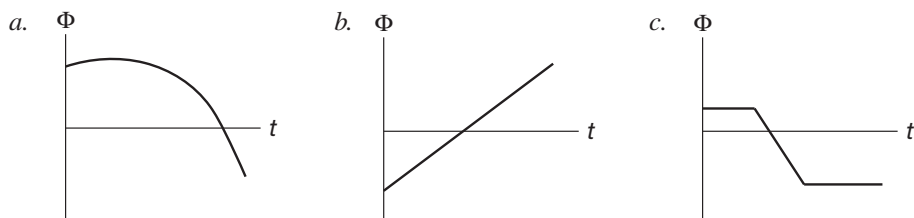
iv. A long time after the switch has been reopened.

No, there will be no external changing magnetic field to induce a current in the wire loop.

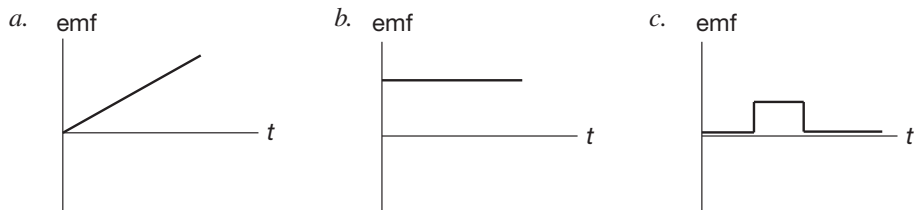
2. An induction stovetop has a smooth surface. When on high, the surface does not feel warm, yet ramen noodles are quickly cooked in a metal bowl. However, when an attempt is made to cook the noodles in a ceramic bowl, the noodles remain cold. Explain how the stovetop works.

The stovetop must generate a changing magnetic field beneath the surface. This changing magnetic field penetrates through the metal (ferromagnetic) pan. The changing magnetic flux through the metal pan induces currents on the metal like it does in a wire. This in turn will generate heat and cook the food.

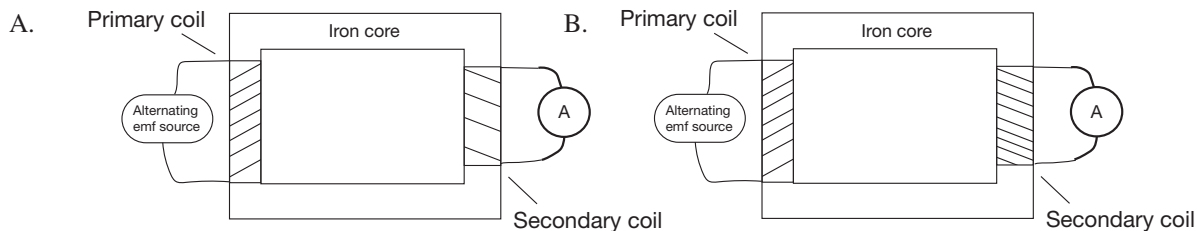
3. The magnetic flux through three different coils is changing as shown in the figures below. For each situation, draw a corresponding graph showing qualitatively how the induced emf changes with time.



Solution:



4. A transformer consists of two coils, each wrapped around an iron core. The core confines the magnetic field produced by the electric current in one coil so that it passes through the second coil without spreading outside. The primary coil in each situation below is connected to identical emf sources.



Which ammeter would register the higher secondary emf output? Explain your reasoning.

The ammeter in situation B would register the higher emf output. If the magnetic field produced by the primary coil completely passes through the secondary coil, then the resulting magnetic flux through the secondary coil would induce a current in that coil. The only difference between the secondary coils in each scenario is the number of turns. When analyzing Equation 1, the more turns in a coil of wire, the larger the induced emf. Since all other variables are kept constant, the secondary coil in situation B must have a higher induced emf because it has more turns than the coil in situation A.

Teacher's Notes *continued*

Reference

AP Physics 1: Algebra-Based and Physics 2: Algebra-Based Curriculum Framework*; The College Board: New York, NY, 2014.

***Electromagnetic Induction—Advanced Inquiry Laboratory Kit* and supporting supplies are available from Flinn Scientific, Inc.**

Catalog No.	Description
AP8005	Electromagnetic Induction—Advanced Inquiry Laboratory Kit
AP5742	Galvanometer, -500 to $+500$ μA

Consult your *Flinn Scientific Catalog/Reference Manual* for current prices.

Fluid Dynamics

AP* Physics 2 Big Ideas 1 and 5, Investigation 2

An Advanced Inquiry Lab

Introduction

Understanding the motion of fluids is important to build a greater understanding of engineering fields such as aerodynamics, hydrodynamics and traffic engineering. Fluid dynamics is used to calculate the forces that act on a flying aircraft, predict weather patterns, and even determine the rate of flow of blood in the body. By exploring the laws of conservation of mass and energy, we can discover the laws that govern fluid dynamics.

Concepts

- Conservation of energy
- Continuity equation
- Bernoulli's law
- Pressure

Background

The concept of pressure is central to the study of fluid dynamics. A fluid exerts pressure in all directions and particularly in a direction perpendicular to any wall with which it makes contact. French physicist, Blaise Pascal (1623–1662), contributed greatly to the principles of hydraulic fluids. When experimenting with hydraulic pistons, he determined that an increase in the pressure of a static, enclosed fluid at one place in the fluid caused a uniform increase in pressure throughout the fluid. Pascal's second law states that the amount of pressure exerted at any given point depends upon the height of the liquid above that particular point. Pressure is expressed as a force over an area (Equation 1).

$$P = \frac{F}{A} \quad \text{Equation 1}$$

When considering fluid flow, it is important to note that liquids are regarded to be almost incompressible. Gases can be compressed due to the large spaces between molecules. The molecules of a liquid are much closer together and cannot be compressed in the same sense. Liquid molecules instead slide past each other. Now consider water flowing in a pipe as shown in Figure 1.

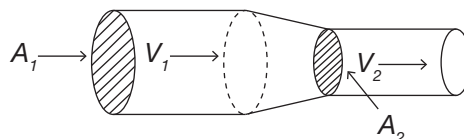


Figure 1.

Since liquids are almost incompressible, the amount of water flowing through cross section A_1 is the same as the amount of water flowing through cross section A_2 . This means that the flow rate through cross section A_1 is the same as the flow rate through cross section A_2 . Flow rate is defined as the volume of fluid that flows through a cross section of a pipe in the time interval $t_2 - t_1$ (Equation 2).

$$Q = \frac{V}{\Delta t} \quad \text{Equation 2}$$

where

Q = flow rate,

V = volume of fluid,

Δt = time interval ($t_2 - t_1$).

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In the narrower section of the pipe, the speed of the water will increase in order to keep the flow rate constant. This is shown in the *continuity equation*:

$$Q_1 = A_1 v_1 = A_2 v_2 = Q_2 \quad \text{Equation 3}$$

where

v_1 = average speed of fluid flowing through cross section of area A_1 ,

v_2 = average speed of fluid flowing through cross section of area A_2 .

Swiss physicist Danile Bernoulli (1700–1782) conducted experiments focusing on the conservation of energy using liquids. He observed that in a segment of pipe with a large diameter, water flowed slowly whereas the same stream of water would flow more quickly through a segment of smaller diameter (as in Figure 1). It was clear that some force had to be acting on the water for it to increase speed. The force for the acceleration could be accounted for by considering pressure differences. If the pressure in the larger diameter segment is greater than the pressure in the smaller diameter segment, then there is a net force from the left (at high pressure) toward the constricted pipe segment on the right (low pressure). Therefore, the liquid accelerates as it enters the constricted segment of the pipe. *Bernoulli's principle* states that the pressure a fluid exerts on a surface decreases as the speed with which the fluid moves across the surface increases. This principle is applied to not only liquids but to all fluids and has as many applications with regard to airflow as it does to liquid flow. To quantify this principle, three assumptions must be made: the fluid is incompressible, the fluid moves without friction, and the flow is streamline (without turbulence). *Bernoulli's equation* is expressed as:

$$P_1 + \frac{1}{2} \rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho g h_2 \quad \text{Equation 4}$$

where

P = pressure, Pa (Pascals),

ρ = fluid density, kg/m³,

v = fluid speed, m/s,

g = acceleration due to gravity, m/s²,

h = depth of the fluid, m.

This equation describes the conservation of energy as applied to fluids; it relates the pressures, speeds and elevations of two points along a single streamline in a fluid. The conservation of energy statement is most apparent when all terms are multiplied by volume. The density of a substance multiplied by its volume equates to mass. The respective terms then become PV (work), $\frac{1}{2}mv^2$ (kinetic energy), and mgh (potential energy).

Experiment Overview

The purpose of this advanced inquiry investigation is to gain an understanding of fluid motion by applying the laws of conservation of energy and mass. An introductory class demonstration of conservation of mass flow is performed by changing the ending cross-sectional area of tubing connected to a faucet. This allows for a class discussion focused on the continuity equation. The guided-inquiry activity explores the relationship between fluid depth in a container and the rate at which the fluid flows from the container.

Teacher Demonstration

Answer *Pre-Lab Question 1* and then observe the demonstration.

Pre-Lab Questions

1. Predict will happen when your teacher blows air underneath the index card “table.”
2. Describe what happened when the teacher blew air underneath the card. What are the forces acting on the card that account for your observations? How does the pressure below the card compare to the pressure above the card?
3. What is the difference between flow rate of a fluid and the speed of the fluid? *Hint*: Consider the units of measurement of each quantity.
4. A boy holding a water gun shoots at his friend standing 5 meters away but misses to the right. At the time of firing, the gun nozzle is 120 cm from the ground. At what speed does the water exit the nozzle?
5. An Olympic swimmer dives to the bottom of a 3-meter deep pool. How much pressure does the diver experience at the bottom of the pool?

Materials

Bottle cap
Bottle, plastic, 1-L
Bucket to catch water
Calipers (optional)
Ruler, metric

Sink or water catch basin
Tape, masking
Tubing connector, $\frac{1}{2}$ " i.d.
Tubing connector, $\frac{3}{8}$ " i.d.
Tubing, rubber/latex

Safety Precautions

Wear safety glasses. Quickly wipe up any spills to prevent injury. Wash hands thoroughly with soap and water before leaving the laboratory. Follow all laboratory safety guidelines.

Introductory Activity

1. Connect the rubber tubing to a sink faucet.
2. Adjust the tubing so that the end where water will exit is at table height (or higher) and fixed parallel to the floor.
3. Place towels along the floor where water might splash.
4. Place a bucket in front of the tubing to collect the exiting stream of water.
5. Insert the smaller tubing connector into the end of the tubing.
6. Measure the inner diameter of the tubing connector. You may use calipers if available.
7. Turn on the faucet so a stream of water falls into the bucket. Mark how far the faucet handle is turned. *Note:* In order to reduce the amount of water spilled, have someone hold the bucket up to the exit nozzle. Then, turn on the faucet gradually while the person holding the bucket walks away with water continuously falling into the bucket until the bucket is able to be placed on the floor.
8. Measure the distance, h , from where the water exits to the floor.
9. Measure the horizontal distance, x , that the stream of water travels until it makes contact with the bottom of the bucket.
10. Turn off the faucet. *Note:* In order to reduce the amount of water spilled, have someone pick up the bucket and follow the stream of water back toward the exit nozzle while the faucet is gradually closed.
11. Repeat steps 5–10 with the larger tubing connector. Be sure to open the faucet handle to the same point used in step 7. Adjust the position of the bucket to collect the water.
12. Wipe up any water off the floor.

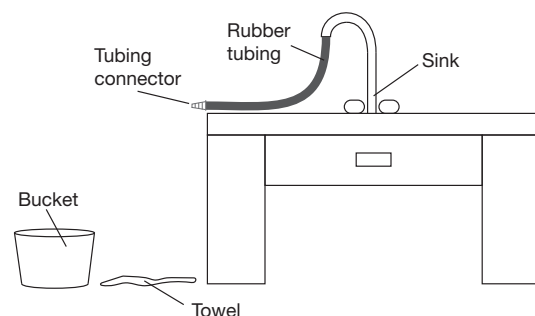


Figure 2.

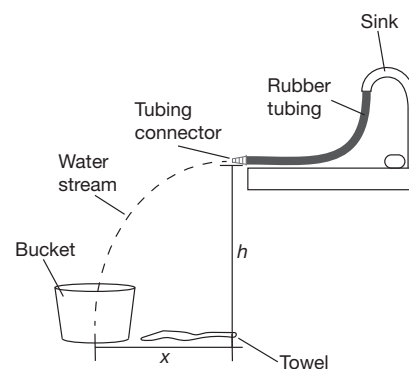


Figure 3.

Analyze the Results

- From the data, calculate the speed of the water exiting each tubing connector with the projectile motion equations.
- Are the flow rate values for the different connectors expected to be the same? Calculate and compare the flow rates when using the smaller connector and the larger connector.
- If the flow rates differ, explain why that may occur.

Guided-Inquiry Design and Procedure

1. Obtain a 1-L bottle that has a hole pierced near the bottom
2. If the 1-L bottle is filled to the top with water, a stream will pour from the pierced hole. What happens to fluid pressure at the exit hole as fluid depth decreases?
3. Choose the initial max height of water in the bottle.
4. Devise a method for determining fluid as water exits the container.
5. Formulate a mathematical technique for determining fluid velocity as it exits the container. *Hint:* Consider how the projectile motion equations were used to calculate fluid velocity in the *Introductory Activity*.
6. With the materials available, devise a procedure for collecting data that can be used to demonstrate the relationship between fluid depth above the exit hole and the rate at which fluid flows from the container. Be sure to gather sufficient values of fluid depth and respective fluid exit velocity in order to provide scientifically accurate data.

Analyze the Results

- Present a graph(s) that represents the data collected.
- From the created graph(s), derive a mathematical relationship between fluid depth above the exit hole and the rate at which water leaves the bottle.
- What are some possible sources of error when collecting data points?

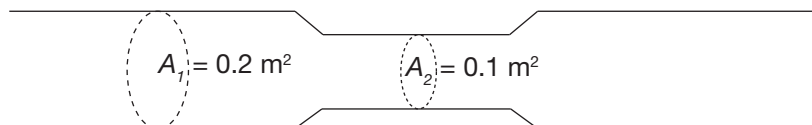
Opportunities for Inquiry

Investigate how changing the density of the fluid affects the results. A salt water mixture may be used for a higher density fluid and an alcohol/water mixture for a lower density fluid.

AP Physics 2 Review Questions***Integrating Content, Inquiry, and Reasoning***

- Consider the same experimental setup as in the *Guided-Inquiry Activity*.
 - Using Bernoulli's equation, derive a mathematical relationship between fluid height in the container and the speed with which the water exits the container.
 - With the relationship derived in Question 1a, do you expect the experimental values to be higher than or lower than the theoretical values? Calculate what the theoretical values of fluid speed should be for each point of fluid depth measured.
 - Give an explanation for any difference in value between the theoretical and experimental fluid speeds.
 - How does the flow rate of water dropping in height throughout the bottle compare to the flow rate of water exiting the hole?
- In your own words, explain how the systems analyzed in the *Introductory Activity* and the *Guided-Inquiry Activity* follow known conservation laws.

- A volume of 2 m^3 of water is flowing in a level, horizontal pipe with a flow rate of $1 \text{ m}^3/\text{s}$. The water flows from a pipe section with a cross sectional area of 0.2 m^2 to a pipe section with a 0.1 m^2 cross sectional area. Before entering the constricted segment of pipe, the water is at a pressure 20 kPa above atmospheric pressure. What is the average acceleration that the volume of water undergoes when flowing from the unconstricted pipe segment to the constricted pipe segment? See Figure 2.

**Figure 2.**

- Far into the future, Mars has been successfully colonized. A farmer on Mars has a personal water tower for which the water level is always kept constant. This water tower is used to feed the farmer's irrigation system. Water from the water tower flows down through a single tube and is held under pressure behind an irrigation system valve (the water is no longer moving at this point). In order to irrigate the field, the water is held at 100 kPa when the valve is closed.
 - How tall must the water tower be in order to irrigate the field? The water pressure in the tower is 1 kPa and the acceleration due to gravity on Mars is 3.8 m/s^2 .
 - Once the valve is opened, how fast does the water flow? The atmospheric pressure on the Martian surface is about 600 pascals.

Teacher's Notes

Investigation 2—Fluid Dynamics

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Part I. Lab Preparation

Materials Included in Kit (for 24 students working in groups of four)

- | | |
|--------------------------|--------------------------------|
| Bottle caps, 6 | Tubing connector, 1/2" i.d., 1 |
| Bottles, plastic, 1 L, 6 | Tubing connector, 3/8" i.d., 1 |
| Rulers, metric 6 | |

Additional Materials Required (for each lab group)

- | | |
|---|---------------------------|
| Butane safety lighter or Bunsen burner | Sink or water catch basin |
| Calipers (optional) | Tape, masking |
| Cork borer, 5 mm i.d. or soldering iron | Tubing, rubber/latex |

Time Required

This laboratory activity can be completed in two 50-minute class periods. It is important to allow time between the *Introductory Activity* and the *Guided-Inquiry Activity* for students to discuss and design the guided-inquiry procedures. Also, all student-designed procedures must be approved for safety before students are allowed to implement them in the lab. *Pre-Lab Questions*, in this case, are required to be completed after a teacher demonstration.

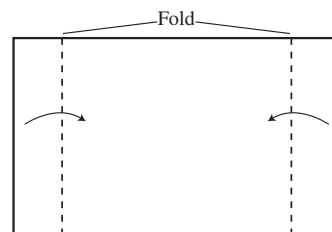


Figure 3.

Teacher Demonstration Instructions

Using a 3" × 5" index card, make 1 cm tabs on the 3" end to make a "table" (see Figures 3 and 4). This and a straw will be needed for the teacher demonstration before pre-lab questions are answered. Set the card on a flat surface and ask students to answer *Pre-lab Question 1*. Then, using a straw, blow a constant fast stream of air underneath the surface of the card. The card will bend down. Students can now complete the remaining *Pre-lab Questions*.

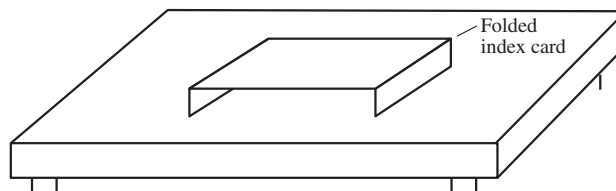


Figure 4.

Teacher's Notes *continued*

Pre-Lab Preparation

To prepare 1-L bottles with small holes:

1. Find a vertical seam on the bottle. Measure 5 cm from the bottom of the bottle and use a marker to place a dot at that location.
2. Using a butane safety lighter, heat the end of a 5 mm i.d. cork borer by holding the end in the flame of the lighter for 20 seconds.
3. Use the cork borer to pierce the bottle at the location of the dot. Care should be taken that the hole is made perpendicular to the bottle so the stream emerges in a horizontal direction. *Note:* The holes can be cut into 6 bottles using the method above (or using the round end of a soldering iron) in under 15 minutes.

Safety Precautions

Use caution when working with hot objects. Quickly wipe up any spills to prevent injury. Wear safety glasses. Wash hands thoroughly with soap and water before leaving the laboratory. Follow all laboratory safety guidelines.

Disposal

All materials may be stored and saved for future use.

Part II. Teacher Guidance

Alignment to AP* Physics 2 Curriculum Framework

Enduring Understandings and Essential Knowledge

Materials have macroscopic properties that result from the arrangement and interactions of the atoms and molecules that make up the material. (1E)

1E1: Matter has a property called density.

The energy of a system is conserved. (5B)

5B10: Bernoulli's equation describes the conservation of energy in fluid flow.

Classically, the mass of a system is conserved. (5F)

5F1: The continuity equation describes conservation of mass flow rate in fluids. Examples should include volume rate of flow and mass flow rate.

Learning Objectives

5B10.1: The student is able to use Bernoulli's equation to make calculations related to a moving fluid.

5B10.2: The student is able to use Bernoulli's equation and/or the relationship between force and pressure to make calculations related to a moving fluid.

5B10.3: The student is able to use Bernoulli's equation and the continuity equation to make calculations related to a moving fluid.

5B10.4: The student is able to construct an explanation of Bernoulli's equation in terms of the conservation of energy.

5F1.1: The student is able to make calculations of quantities related to flow of a fluid, using mass conservation principles (the continuity equation).

Science Practices

1.2 The student can describe representations and models of natural or man-made phenomena and systems in the domain.

1.4 The student can use representations and models to analyze situations or solve problems qualitatively and quantitatively.

2.1 The student can justify the selection of a mathematical routine to solve problems.

2.2 The student can apply mathematical routines to quantities that describe natural phenomena

Teacher's Notes *continued*

- 3.1 The student can pose scientific questions.
- 4.1 The student can justify the selection of the kind of data needed to answer a scientific question.
- 4.2 The student can design a plan for collecting data to answer a particular scientific question.
- 4.3 The student can collect data to answer a particular scientific question.
- 5.1 The student can analyze data to identify patterns and relationships.
- 5.2 The student can refine observations and measurements based on data analysis.
- 6.1 The student can justify claims with evidence.
- 6.2 The student can construct explanations of phenomena based on evidence produced through scientific practices.
- 6.4 The student can make claims and predictions about natural phenomena based on scientific theories and models.
- 7.2 The student can connect concepts in and across domain(s) to generalize or extrapolate in and/or across enduring understanding and/or big ideas.

Lab Hints

- In the *Introductory Activity*, the use of Teflon tape can improve the fit between the tubing and the faucet and reduce the amount of leaks.
- When preparing the 1-L bottles, take care to create round holes with smooth edges to reduce the effects of turbulent flow. Ensure that the cork borer is hot enough so that a clean hole is made, if the end of the cork borer does not immediately go through the bottle, heat it another 20 seconds.

Teaching Tips

- Blowing air over a sheet of paper is a quick demonstration that students could do to further familiarize themselves with Bernoulli's principle.
- In the *Guided-Inquiry Activity*, students may choose to directly measure fluid depth vs. time. This data can still be used to come to the same conclusion about the relationship between flow rate and fluid depth but requires further analysis than the fluid depth vs. fluid velocity method.
- If students opt to study fluid height in the container as a function of time, it is important to guide them away from the confusion that velocity of the water level as it drops in height is the same as the velocity of the water as it exits the hole.
- Giving different groups bottles with different sized holes would add another element of analysis to the investigation.

Part III. Sample Data, Results and Analysis

Answers to Pre-Lab Questions *(Student answers will vary.)*

1. Predict will happen when your teacher blows air underneath the index card "table."
Student answers will vary.
2. Describe what happened when the teacher blew air underneath the index card. What are the forces acting on the card that account for your observations? How does the pressure below the card compare to the pressure above the card?
The card bent down. The forces acting on the card are gravity, air pressure up, and air pressure down. The air pressure below the card is lower than the pressure above the card. The higher pressure air moves towards the lower pressure air, causing the card to bend down.
3. What is the difference between flow rate of a fluid and the speed of the fluid? *Hint: Consider the units of measurement of each quantity.*
Flow rate is the movement of a unit of volume of a fluid per unit of time. Speed of fluid is a unit of distance pre unit of time. A flow rate could be measured in m^3/s whereas a fluid speed is measured in m/s .

Teacher's Notes *continued*

4. A boy holding a water gun shoots at his friend standing 5 meters away but misses to the right. At the time of firing, the gun nozzle is 120 cm from the ground. At what speed does the water exit the nozzle?

$$h = 1.2 \text{ m}$$

$$g = 9.8 \text{ m/s}^2$$

$$h = \frac{1}{2}gt^2$$

$$t = \sqrt{2h/g}$$

$$t = \sqrt{2 \times (1.2)/(9.8)}$$

$$t = 0.49 \text{ seconds}$$

$$\text{Speed} = x/t$$

$$x = 5 \text{ m}$$

$$t = 0.49 \text{ s}$$

$$\text{Speed} = 5/0.49$$

$$\text{Speed} = 10.1 \text{ m/s}$$

5. An Olympic swimmer dives to the bottom of a 3 meter deep pool. How much pressure does the diver experience at the bottom of the pool?

$$\text{Pressure} = \rho gh = (1000 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(3 \text{ m}) = 29.4 \text{ kPa}$$

Sample Data and Discussion for Introductory Activity

- From the data, calculate the speed of the water exiting each tubing connector with the projectile motion equations.

Answers will vary. To calculate the speed of water, first measure the distance, h, from the nozzle to the floor. Calculate the time it takes for the water to fall to floor with $t = \sqrt{2h/g}$ where g is the acceleration due to gravity. Using the t calculated, measure the speed of the exiting water with x/t with x being the horizontal distance the water traveled before impacting with the floor.

- Are the flow rate values for the different connectors expected to be the same? Calculate and compare the flow rates when using the smaller connector and the larger connector.

Yes, the flow rates are expected to be the same due to the conservation of mass. To calculate the flow rate, first measure the inner diameter of the respective connector. Then divide by 2 to get the radius and use πr^2 to calculate area. Flow rate is area multiplied by speed of fluid.

Connector	Area	Speed	Flow Rate
1	0.000095 m ²	0.66 m/s	0.0000627 m ³ /s
2	0.0000567 m ²	1.23 m/s	0.0000697 m ³ /s

- If the flow rates differ, explain why that may occur.

Answers will vary. The flow rates are very close in value but differ slightly. A reason for this difference could be due to the faucet not being open to exactly the same location for each connector tested. Therefore, the flow rates differed slightly.

Sample Data and Discussion for Guided-Inquiry Activity

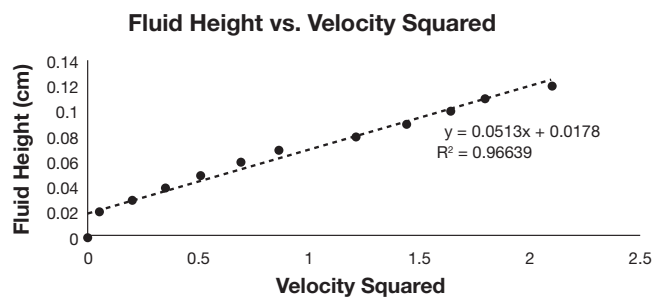
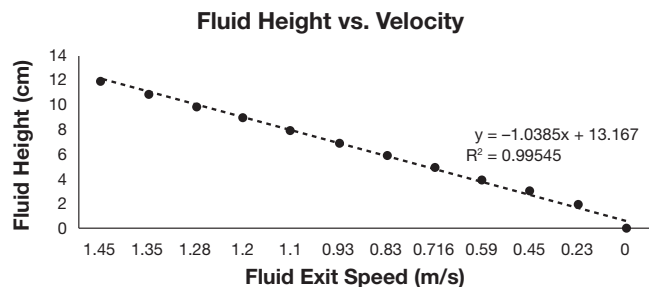
- Obtain a 1-L bottle that has a hole pierced near the bottom.
- Choose the max height of water on the bottle.
- With the materials available, devise a procedure for collecting data that can be used to demonstrate the relationship between fluid depth above the exit hole and the rate at which fluid flows from the container.

One method students could use is to measure fluid depth vs. velocity. Using a vertical piece of masking tape, create a marked scale on the outside of the bottle. Set the bottle on the end of an elevated surface and record the horizontal distance traveled by the stream from the exit hole to the container on the floor for different fluid depths of water. One could use masking tape to measure spots on the floor that correspond with tick marks on the vertical scale on the bottle. The exit velocity can then be calculated with the projectile motion equation and the data gathered is that of fluid depth and exit velocity.

Teacher's Notes *continued*

Analyze the Results

- Present a graph(s) that represents the data collected.



- From the created graph(s), derive a mathematical relationship between fluid depth above the exit hole and the rate at which water leaves the bottle.

The equation produced from the graph of fluid height vs velocity squared gives us an equation of $h = 0.0513v^2$. When solving for v : $v = \sqrt{(h/0.051)}$ which allows for the conclusion that the rate at which waters flows from the container is proportional to the square root of the water depth above the exit hole.

- What are some possible sources of error when collecting data points?

Possible experimental error may arise from imprecise measurements due expected systematic error (e.g. delayed reaction times, non-uniform tick mark distances, etc). Another source of error leading to losses in kinetic energy of the fluid may arise from slight turbulence at the exit point due to the perforated hole not being smooth enough.

Answers to AP Physics 2 Review Questions

- Consider the same experimental setup as in the *Guided-Inquiry Activity*.

- Using Bernoulli's equation, derive a mathematical relationship between fluid height in the container and the speed with which the water exits the container.

$P_1 + \frac{1}{2}\rho v_1^2 + \rho gh_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho gh_2$; here P_1 and P_2 are atmospheric pressure and can be cancelled out. Further, v_1 is zero and the second term on the left side can be cancelled out. Likewise, h_2 is zero (at the position of the hole) and the third term on the right side can be cancelled out. We are left with $\rho gh_1 = \frac{1}{2}\rho v_2^2$. The densities cancel out.

When solving for v_2 , we get $v_2 = \sqrt{2gh_1}$

- With the relationship derived in Question 1a, do you expect the experimental values to be higher than or lower than the theoretical values? Calculate what the theoretical values of fluid speed should be for each point of fluid depth measured.

Student answers will vary. Experimental values are expected to be lower than theoretical values due to losses in energy from friction and turbulence. Theoretical values of fluid speed can be calculated using the expression derived in 1a. Use the height for each respective fluid depth data point to calculate the respective theoretical fluid speed.

- Give an explanation for any differences in value between the theoretical and experimental fluid speeds.

The theoretical values are higher than the experimental data because in reality you lose energy due to friction and turbulence effects, which results in lower expected speeds.

- How does the flow rate of water dropping in height throughout the bottle compare to the flow rate of water exiting the hole?

The flow rates are the same due to conservation of mass as portrayed in the continuity equation.

Teacher's Notes *continued*

2. In your own words, explain how the systems analyzed in the *Introductory Activity* and the *Guided-Inquiry Activity* follow known conservation laws.

The investigation conducted in the Introductory Activity was a demonstration of conservation of energy, and more specifically, conservation of mass. This is because flow rates are constant due to conservation of mass flow, which is observed in the activity. The experiment in the Guided-Inquiry Activity follows the law of conservation of energy because potential energy due to gravity is completely converted to kinetic energy at the exit hole. Bernoulli's equation is a statement of conservation of energy in fluids, and when used to calculate expected values, it confirmed that energy was conserved.

3. A volume of 2 m^3 of water is flowing in a level, horizontal pipe with a flow rate of $1 \text{ m}^3/\text{s}$. The water flows from a pipe section with a cross sectional area of 0.2 m^2 to a pipe section with a 0.1 m^2 cross sectional area (see Figure 2). Before entering the constricted segment of pipe, the water is at a pressure 20 kPa above atmospheric pressure. What is the average acceleration that the volume of water undergoes when flowing from the unconstricted pipe segment to the constricted pipe segment?

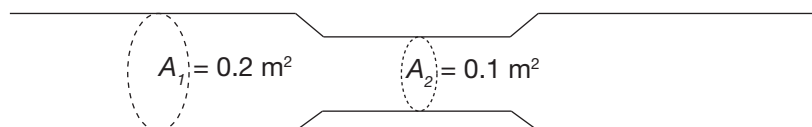


Figure 2.

$P_1 = 121,325 \text{ pascals}$. Volume of water $= 2 \text{ m}^3$. $A_1 = 0.2 \text{ m}^2$. $A_2 = 0.1 \text{ m}^2$. Flow rate $Q = 1 \text{ m}^3/\text{s}$.

Acceleration is found from $P_1 A_1 - P_2 A_2 = F_1 - F_2 = F_{\text{net}} = ma$. a is acceleration. Mass of volume is density of water times volume $= 1000 \text{ kg/m}^3 \times 2 \text{ m}^3 = 2000 \text{ kg}$.

P_2 is found using Bernoulli's equation: $P_2 = P_1 + \frac{1}{2}\rho(v_1^2 - v_2^2)$. (Due to the level fluid flow the terms containing fluid height are eliminated). The speeds v_1 and v_2 can be calculated using the continuity equation where $Q = A_1 v_1 = A_2 v_2$, which is $1 \text{ m}^3/\text{s} = (0.2 \text{ m}^2)v_1 = (0.1 \text{ m}^2)v_2$. $v_1 = 5 \text{ m/s}$ and $v_2 = 10 \text{ m/s}$. When using these values in Bernoulli's equation, P_2 is found to be $83,825 \text{ pascals}$. F_{net} can then be calculated to be $15,882.5 \text{ N}$. Equating F_{net} to ma and solving for a gives an acceleration of 7.94 m/s^2 .

4. Far into the future, Mars has been successfully colonized. A farmer on Mars has a personal water tower for which the water level is always kept constant. This water tower is used to feed the farmer's irrigation system. Water from the water tower flows down through a single tube and is held under pressure behind an irrigation system valve (the water is no longer moving at this point). In order to irrigate the field, the water is held at 100 kPa when the valve is closed.

- a. How tall must the water tower be in order to irrigate the field? The water pressure in the tower is 1 kPa and the acceleration due to gravity on Mars is 3.8 m/s^2 .

Using Bernoulli's equation: $1,000 \text{ pascals} + 0 + \rho(3.8 \text{ m/s}^2)h = 100,000 \text{ pascals} + 0 + 0$.

$h = 26.05 \text{ meters}$.

- b. Once the valve is opened, how fast does the water flow? The atmospheric pressure on the Martian surface is about 600 pascals .

$100,000 \text{ pascals} = 600 \text{ pascals} + \frac{1}{2}\rho v_2^2 + 0$.

$v = 14.1 \text{ m/s}$.

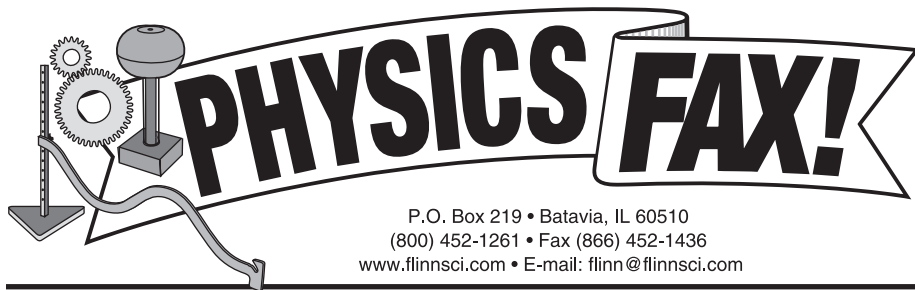
Reference

AP Physics 1: Algebra-Based and Physics 2: Algebra-Based Curriculum Framework*; The College Board: New York, NY, 2014.

Fluid Dynamics—Advanced Inquiry Laboratory Kit and supporting supplies are available from Flinn Scientific, Inc.

Catalog No.	Description
AP7996	Fluid Dynamics—Advanced Inquiry Laboratory Kit
AP8326	Cork Borers, Set of 6

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Speed of Sound

AP* Physics 1 Big Idea 6, Investigation 14

An Advanced Inquiry Lab

Introduction

Lightning flashes and we see it immediately, then a few seconds later we hear a thunderclap. Since light travels faster than sound, counting the seconds between the lightning and thunder gives us an approximation of how far away the lightning flashed. Just how fast does sound travel? How can the speed of sound be measured?

Concepts

- Reflection
- Frequency and wavelength
- Sound waves
- Standing waves
- Resonance
- Speed of Sound

Background

Sound is a mechanical wave created by the vibrations of material objects. A mechanical wave requires a medium in order to propagate. Sound propagates by pushing molecules back and forth. If no molecules are present, such as in a vacuum, sound will not travel.

As a sound wave travels through a medium, it may encounter a boundary, or interface of two different media. Upon reaching a boundary, the energy of the sound wave may be reflected, transmitted, or absorbed—often a combination of the three. An echo is an example of a sound wave that is mostly reflected. The difference in time, t , between the creation of the sound wave and the perception of the reflected sound is an indication of the distance to the boundary. If the boundary is a solid, flat object such as a wall, and the distance, d , from the origin of the sound to the boundary is known, the speed of sound can be calculated using the equation, $v = 2d/t$.

Reflection of sound waves also comes into play in many musical instruments. Woodwind and brass instruments produce sound as air is vibrated in an air column. The length of the air column is varied to change the pitch of the sound produced. The vibrating source produces a sound wave that moves through the air column and is ultimately reflected back toward the vibrational source. The sound produced is the loudest when the air column is in resonance with the vibrational source.

How does resonance occur? The sound wave in the air column consists of alternating high- and low-pressure variations. The sound wave is ultimately reflected back toward the vibrational source. It is either reflected back off a closed end of the column or as a low-pressure reflection off the open end of the column. If the reflected wave reaches the vibrational source at the same moment another wave is produced, then the incident and reflected waves constructively interfere with each other. A standing wave is produced, creating resonance. A standing wave in a column of air can be represented by a sine wave in either an open tube (open at both ends) or a closed tube (closed at one end). A standing wave in an open tube has an antinode at each end, which is the boundary between the air in the enclosed region of the tube and the air outside the tube. In a closed tube, the closed end acts as a fixed point, which prevents movement, and the reflected wave will be inverted. Therefore, the closed end of the tube will always contain a node (see Figure 1 on page 2). Because the amplitude is largest at an antinode, the sound is the loudest at this point.

Standing wave patterns are only created at an instrument's natural frequencies, known as harmonic frequencies. The fundamental frequency (f_1) corresponds to the first harmonic. Each harmonic is an integral multiple of the fundamental frequency ($f_n = n f_1$). As the frequency is increased, additional resonance lengths are found at half-wavelength intervals. An open tube resonates when its length is an even number of quarter wavelengths (2, 4, 6, etc.), or every half-wavelength. A closed tube resonates when its length is an odd number of quarter wavelengths (1, 3, 5, etc.).

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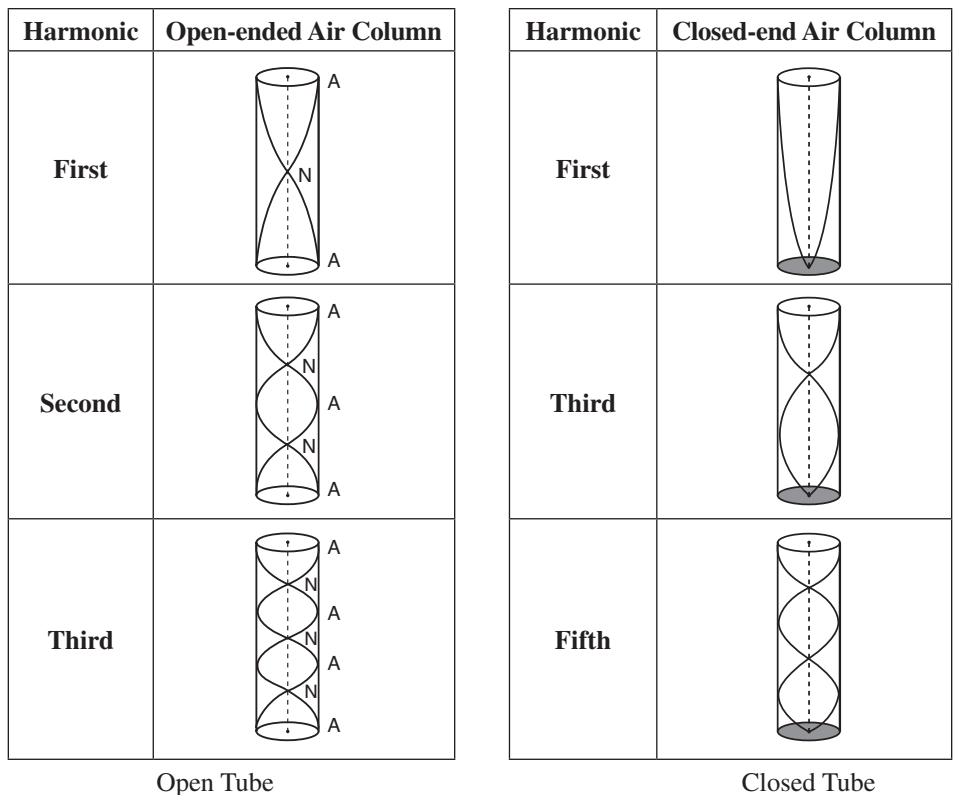


Figure 1.

The speed of sound can be calculated using Equation 1 below. According to Equation 1, if the frequency (f) and wavelength (λ) of a sound wave are known, the speed of sound (v) can easily be calculated by multiplying the two values together.

$$v = f\lambda \quad \text{Equation 1}$$

where

v is the speed (m/s)

f is the frequency (Hz)

λ is the wavelength (m)

The speed of sound is not a constant value and varies depending on the medium in which it travels. At 0 °C the accepted value for the speed of sound in dry air is 331.4 m/s. As the temperature of air increases, the speed of sound also increases because molecules in hot air move more rapidly and collide more often than molecules in cool air. Equation 2 shows the temperature dependence factor for the speed of sound in dry air.

$$v = 331.4 \text{ m/s} + \frac{0.6 \text{ m/s}}{^{\circ}\text{C}} \times T \quad \text{Equation 2}$$

where

T is the temperature in degrees Celsius

Experiment Overview

The purpose of this advanced inquiry lab is to investigate methods for measuring the speed of sound in air. The investigation begins with an introductory cooperative class activity to determine the speed of sound in air using echoes. The experimental wave speed will be compared to the theoretical value. In the guided-inquiry section of the lab, an experiment to measure the speed of sound in a closed-end air column will be designed using tuning forks of known frequencies.

Pre-Lab Questions

1. Why does the equation for the velocity of sound in air using an echo include a factor of 2 for the distance?
2. A hiker stands at the entrance to a canyon and shouts. The echo is heard by the hiker 0.6 seconds later. The temperature of the air in the canyon is 19 °C. How far is the hiker from the canyon wall?
3. Using Figure 1 from the *Background* section as a guide, complete the charts below for each harmonic of an air column in open and closed tubes, respectively, where L is the length of the tube. The first harmonic has been completed.

Open Tube		
Harmonic	No. of Waves in Tube	Length–Wavelength Relationship
1	1/2	$\lambda_1 = (2/1) \times L$
2		
3		

Closed Tube		
Harmonic	No. of Waves in Tube	Length–Wavelength Relationship
1	1/4	$\lambda_1 = (4/1) \times L$
3		
5		

4. Use Equation 1 from the *Background* section and the information from the chart above to determine the fundamental mathematical relationship of the natural frequency (f_n) of an open tube and of a closed tube, respectively, to the speed of sound in air (v) and the length of the tube (L).

Materials

Clamp holders, 2	Support stand
Clamps, universal extension, 2	Tape measure, metric
Graduated cylinder or beaker, 250-mL	Thermometer, Celsius
Petroleum jelly (optional)	Timer or stopwatch
Plastic tube, clear, 1"-diameter, 2 ft	Tuning forks
PVC tube, white, 1/2"-diameter, 2 ft	Tuning fork activator
Rubber stopper, #5	Water, 200 mL
Ruler, metric	Wood blocks, 2 hinged together with tape

Safety Precautions

Use caution when clapping the wood blocks together so fingers will not get pinched. Wipe up any water spills immediately. Please follow all laboratory safety guidelines.

Introductory Activity (Cooperative Class Activity)

Read the entire procedure before beginning. Construct an appropriate data table to record measurements and the results of calculations.

1. Locate a spot outdoors at least 20 m away from a large, flat wall with nothing between the spot and the wall that might interfere with sound wave propagation. A greater distance, 50 m or more, is preferable.
2. Record the ambient temperature.
3. One person should stand at the measured spot with the hinged wood blocks and clap them together once. Clap the blocks with the broad side of the blocks facing the wall.
4. Several other people should sit or stand by the “clapper” the same distance away from the wall and listen for the echo.
5. One or more persons stand nearby, each with a stopwatch or timer.
6. The clapper should repeat step 3 until the time between the clap and when the echo is heard is fairly certain. The others listening for the echo can signal when the echo is heard.
7. The clapper now claps the blocks together repeatedly so that each subsequent clap is created at the same time the echo of

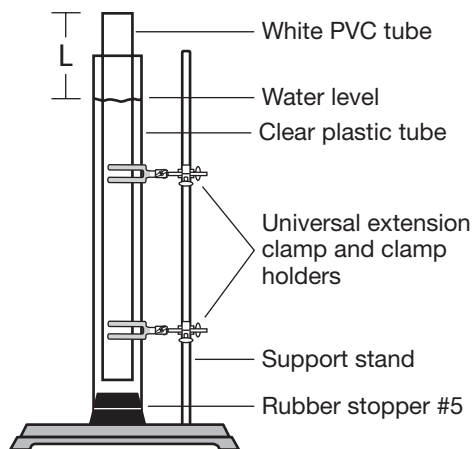
the previous clap is heard. The listeners can signal when synchronization of the claps and echoes has been achieved.

- When the claps are in synchronization with the echoes, the clapper counts down in time with the claps, “3, 2, 1, zero,” and immediately counts back up to a set number of claps (10 to 30 claps is usually sufficient for timing).
- At “zero” the timers start timing and stop when the set number of claps is reached.
- Repeat steps 7–9 several times to determine precision.

Guided-Inquiry Design and Procedure

Form a working group with other students and discuss the following questions.

- Review the *Introductory Activity*.
 - Calculate the speed of sound in air using the timing of the echoes.
 - Determine the theoretical speed of sound for the location by factoring in the effect of temperature.
 - Calculate the percent error between the theoretical and experimental values for the speed of sound.
 - Identify sources of error that may account for discrepancies between the calculated and theoretical speed of sound.
- Vibrating tuning forks produce sound waves at known frequencies. Each tuning fork vibrates most strongly at its fundamental frequency (first harmonic), which is stamped on the instrument. If a vibrating tuning fork were used to drive a sound wave in a closed tube, what would be the relationship of the frequency to the length of the air column and the speed of the sound wave when resonance is achieved?
- Assume the length of the tube described in question 2 can be varied. How would you know when resonance is achieved?
- Predict how the length of the air column in a closed tube will vary as the frequency of the tuning fork changes.
- How might the setup pictured below be used to create resonance at different frequencies?



- Write a step-by-step procedure for measuring the speed of sound in a closed tube using tuning forks. Construct a data table that clearly shows the data that will be collected and the measurements that will be made.
- How can the data be presented graphically to show a linear relationship between the variables and to calculate the speed of sound?
- Calculate the speed of sound in the closed tube. Calculate the theoretical speed of sound for the air in the tube and determine the percent error. Identify sources of error in the experiment.

Opportunities for Inquiry

Design and create a musical instrument with a range of at least one octave that can play a simple tune.

AP Physics 1 Review Questions

Integrating Content, Inquiry and Reasoning

1. Compare the accuracy of the two methods for determining the speed of sound in air—timing echoes and achieving resonance in a closed tube. How might the accuracy of each method be improved?
2. How would the slope of graphed data for the guided-inquiry activity change if hot water were used instead of room temperature tap water?
3. The PVC tube used in the guided-inquiry activity is 61 cm long. Of the eight tuning forks available in this activity, what is the lowest frequency tuning fork that would be able to resonate in the PVC tube at the next higher harmonic? Explain your reasoning.
4. A boat on Lake Michigan is outfitted with an echo sounder—a type of sonar unit that sends a wave pulse downward and then detects the pulse reflected from the lake bottom. If the time between the incident pulse and the detection of the reflected pulse is 0.27 seconds, how deep is the lake bottom, assuming the speed of sound in the water is 1500 m/s?
5. Most orchestras tune to concert pitch, which is 440 Hz.
 - a. A clarinet is considered a closed tube, since the musician's mouth covers the vibrating reed on the mouthpiece. What is the length of the air column in the clarinet when it is in tune at concert pitch? Assume the temperature of the room is 20 °C.
 - b. A flute is an open tube. What is the length of the air column in the flute when it is in tune at concert pitch in the same room?
 - c. A piccolo is like a small flute, just 32 cm long. Can the piccolo tune to the fundamental frequency of concert pitch in the same room? If not, how can a piccolo tune with the orchestra?

Teacher's Notes

Investigation 14—Speed of Sound

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Part I. Lab Preparation

Materials Included in Kit (for 8 student groups)

Plastic tubes, clear, 1"-diameter, 2 ft
PVC tubes, white, ½"-diameter, 2 ft
Rubber stoppers, solid, #5

Tuning forks, set of 8
Wood blocks, 6 ⅛" × 2 ½" × 1"

Additional Materials Needed (for each lab group)

Clamp holders, 2
Clamps, universal extension, 2
Graduated cylinder or beaker, 250-mL
Petroleum jelly (optional)
Ruler, metric
Support stand

Tape, duct or masking*
Tape measure, metric
Timer or stopwatch
Tuning fork activator or rubber mallet
Water, 200 mL

*For Pre-Lab Preparation. See details below.

Pre-Lab Preparation

Place the two wood blocks together and securely tape together the short ends of the blocks to form a hinge.

Time Required

This laboratory activity can be completed in two 50-minute class periods. It is important to allow time between the *Introductory Activity* and the *Guided-Inquiry Activity* for students to discuss and design the guided-inquiry procedures. Also, all student-designed procedures must be approved for safety before students are allowed to implement them in the lab. *Pre-Lab Questions* may be completed before lab begins the first day.

Teacher's Notes *continued*

Safety Precautions

Use caution when clapping the wood blocks together so fingers will not get pinched. Wipe up any water spills immediately. Please follow all laboratory safety guidelines.

Disposal

The water in the tube setup may be placed down the drain. All materials may be dried and saved for future use.

Part II. Teacher Guidance

Alignment to AP Physics 1 Curriculum Framework

Enduring Understandings and Essential Knowledge

A wave is a traveling disturbance that transfers energy and momentum. (6A)

6A2: For propagation, mechanical waves require a medium, while electromagnetic waves do not require a physical medium. Examples should include light traveling through a vacuum and sound not traveling through a vacuum.

6A4: Classically, the energy carried by a wave depends upon and increases with amplitude. Examples should include sound waves.

A periodic wave is one that repeats as a function of both time and position and can be described by its amplitude, frequency, wavelength, speed, and energy. (6B)

6B2: For a periodic wave, the wavelength is the repeat distance of the wave.

6B4: For a periodic wave, wavelength is the ratio of speed over frequency.

Interference and superposition lead to standing waves and beats. (6D)

6D3: Standing waves are the result of the addition of incident and reflected waves that are confined to a region and have nodes and antinodes. Examples should include waves on a fixed length of string, and sound waves in both closed and open tubes.

Learning Objectives

6A2.1: The student is able to describe sound in terms of transfer of energy and momentum in a medium and relate the concepts to everyday examples.

6A4.1: The student is able to explain and/or predict qualitatively how the energy carried by a sound wave relates to the amplitude of the wave, and/or apply this concept to a real-world example.

6B4.1: The student is able to design an experiment to determine the relationship between periodic wave speed, wavelength, and frequency and relate these concepts to everyday examples.

6D3.1: The student is able to refine a scientific question related to standing waves and design a detailed plan for the experiment that can be conducted to examine the phenomenon qualitatively or quantitatively.

6D3.2: The student is able to predict properties of standing waves that result from the addition of incident and reflected waves that are confined to a region and have nodes and antinodes.

6D3.3: The student is able to plan data collection strategies, predict the outcome based on the relationship under test, perform data analysis, evaluate evidence compared to the prediction, explain any discrepancy and, if necessary, revise the relationship among variables responsible for establishing standing waves on a string or in a column of air.

Science Practices

3.2 The student can refine scientific questions.

4.1 The student can justify the selection of the kind of data needed to answer a particular scientific question.

4.2 The student can design a plan for collecting data to answer a particular scientific question.

5.1 The student can analyze data to identify patterns or relationships.

5.3 The student can evaluate the evidence provided by data sets in relation to a particular scientific question.

6.4 The student can make claims and predictions about natural phenomena based on scientific theories and models.

7.2 The student can connect concepts in and across domain(s) to generalize or extrapolate in and/or across enduring understandings and/or big ideas.

Teacher's Notes *continued*

Lab Hints

- In order for the human ear to perceive an echo, the time lapse must be greater than 0.1 second. Otherwise a reverberation is heard. The greater the distance from the reflecting surface, the easier it is to time the claps. However, the echo will be fainter with greater distance.
- If the *Introductory Activity* is not feasible due to lack of ideal conditions, the echo method may also be conducted by other means. Use a computer interface system such as Vernier LabQuest™ (Flinn Catalog No. TC1561) with a microphone (Flinn Catalog No. TC2322) to record the initial sound and its echo. An online program, Audacity, is available for free download at audacity.sourceforge.net (accessed June 2014). The precision timing of either of these options allows for a much shorter distance between the sound origin and the echo.
- You may need to model the proper way to strike a tuning fork with an activator or a rubber mallet.
- If time permits, the tuning forks may be rotated among the student groups. Otherwise, each group may use a different tuning fork and share the data.

Teaching Tip

- A systematic error in the *Guided-Inquiry Activity* is the antinode of the standing wave at the open end of the air column is not exactly at the tube opening, but just beyond the end of the tube. The actual length differs from the measured length by a factor of 0.6 times the inner diameter of the tube. Factoring in this “end correction” is an option.

Part III. Sample Data, Results and Analysis

Answers to Pre-Lab Questions

1. Why does the equation for the velocity of sound in air using an echo include a factor of 2 for the distance?
Since the sound travels from the origin to the boundary and back, the total distance traveled is double the distance from the origin to the boundary.
2. A hiker stands at the entrance to a canyon and shouts. The echo is heard by the hiker 0.6 seconds later. The temperature of the air in the canyon is 19 °C. How far is the hiker from the canyon wall?

The velocity of sound at 19 °C is $331.4 \text{ m/s} + \frac{0.6 \text{ m/s}}{^{\circ}\text{C}} \times 19^{\circ}\text{C} = 342.8 \text{ m/s}$.

Solving for distance from the hiker to the canyon wall:

$$d = \frac{342.8 \text{ m/s} \times 0.6 \text{ s}}{2} = 103 \text{ m}$$

3. Using Figure 1 from the *Background* section as a guide, complete the charts below for each harmonic of an air column in open and closed tubes, respectively, where L is the length of the air column. The first harmonic has been completed.

Open Tube		
Harmonic	No. of Waves in Tube	Length–Wavelength Relationship
1 (f_1)	1/2	$\lambda_1 = (2/1) \times L$
2 (f_2)	2/2 or 1	$\lambda_2 = (2/2) \times L$
3 (f_3)	3/2	$\lambda_3 = (2/3) \times L$

Closed Tube		
Harmonic	No. of Waves in Tube	Length–Wavelength Relationship
1 (f_1)	1/4	$\lambda_1 = (4/1) \times L$
3 (f_3)	3/4	$\lambda_3 = (4/3) \times L$
5 (f_5)	5/4	$\lambda_5 = (4/5) \times L$

4. Use Equation 1 from the *Background* section and the information from the chart above to determine the fundamental mathematical relationship of the natural frequency (f_n) of an open tube and of a closed tube, respectively, to the speed of sound in air (v) and the length of the tube (L).

Open Tube: $f_n = n \frac{v}{2L}$, where n is an integer representing the number of half wavelengths of the standing wave in the tube.

Closed Tube: $f_n = n \frac{v}{4L}$, where n is an odd-numbered integer representing the number of quarter wavelengths of the standing wave in the tube.

Teacher's Notes *continued*

Sample Data for Introductory Activity

Distance to wall: 45 m

Temperature: 23.5 °C

Trial	Time for 20 Claps (s)	Time for 1 Echo (s)
1	5.40	0.27
2	4.88	0.24
3	4.97	0.25
4	4.72	0.24
5	5.16	0.26
6	4.93	0.25
7	5.31	0.27
8	5.09	0.26
9	5.24	0.26
10	5.21	0.26
Avg.	5.07	0.26

Answers to Guided-Inquiry Discussion Questions

1. Review the *Introductory Activity*.

a. Calculate the speed of sound in air using the timing of the echoes.

$$v = 2(45 \text{ m}/0.26 \text{ s}) + \frac{0.6 \text{ m/s}}{^{\circ}\text{C}} \times 23.5 \text{ }^{\circ}\text{C} = 346 \text{ m/s} + 14 \text{ m/s} = 360 \text{ m/s}$$

b. Determine the theoretical speed of sound for the location by factoring in the effect of temperature.

$$v = 331.4 \text{ m/s} + \frac{0.6 \text{ m/s}}{^{\circ}\text{C}} \times 23.5 \text{ }^{\circ}\text{C} = 346 \text{ m/s}$$

c. Calculate the percent error between the theoretical and experimental values for the speed of sound.

$$\frac{360 \text{ m/s} - 346 \text{ m/s}}{346 \text{ m/s}} = 4\% \text{ error}$$

d. Identify sources of error that may account for discrepancies between the calculated and theoretical speed of sound.

The relative humidity of the air may be a factor in how fast the sound waves travel. This is a systematic error that pertains to the conditions of the experiment. One could research a method for measuring and factoring in relative humidity on the speed of sound. Accurate timing with a stopwatch is difficult, and at times the consistency of the cadence of the claps was questionable. The effects that timing errors may have on the experimental results can be mitigated by conducting a large number of trials, employing several timekeepers per trial, and eliminating clear outliers from data sets.

Teacher's Notes *continued*

2. Vibrating tuning forks produce sound waves at known frequencies. Each tuning fork vibrates most strongly at its fundamental frequency (first harmonic), which is stamped on the instrument. If a vibrating tuning fork were used to drive a sound wave in a closed tube, what would be the relationship of the frequency to the length of the air column and the speed of the sound wave when resonance is achieved?

For a closed tube, the first harmonic occurs at a wavelength equal to 4 times the length of the vibrating column of air, $\lambda = 4L$. Substituting the relationship between frequency and wavelength from Equation 1 gives $v/f = 4L$ or $f = v/4L$.

3. Assume the length of the tube described in question 2 can be varied. How would you know when resonance is achieved?

When the tube is the correct length, the sound from the tuning fork will be loudest. At this point, the standing wave of the air column in the tube is resonating with the natural frequency of the tube.

4. Predict how the length of the air column in a closed tube will vary as the frequency of the tuning fork changes.

The frequency of the tuning fork and the length of the air column are indirectly proportional. As the frequency of the tuning fork increases, the length of the air column will decrease.

5. How might the setup pictured be used to create resonance at different frequencies?

The water in the tube creates a closed end. The length of the air column is from the open end of the tube to the closed end, or the level of the water. By raising the inner tube, the level of the water changes, thus changing the length of the air column in the tube. If a sound wave were driven into the tube, the tube could be raised or lowered until the sound was loudest, indicating resonance had been achieved. The amount the tube is raised or lowered can be adjusted for different frequencies.

6. Write a step-by-step procedure for measuring the speed of sound in a closed tube using tuning forks. Construct a data table that clearly shows the data that will be collected and the measurements that will be made.

7. How can the data be presented graphically to show a linear relationship between the variables and to calculate the speed of sound?

Since an inverse relationship exists between frequency and the length of the air column, a graph of frequency versus $1/L$ should be presented. Since $f = v/4L$, the speed of sound can be calculated by multiplying the slope of the graph by 4.

8. Calculate the speed of sound in the closed tube. Calculate the theoretical speed of sound for the air in the tube and determine the percent error. Identify sources of error in the experiment.

See Sample Data and Graph. Since $f = 86.073/L$ and $f = v/4L$, $v = 4(86.073) = 344.3$ m/s. The theoretical speed at the measured temperature is:

$$331.4 + \frac{0.6 \text{ m/s}}{^{\circ}\text{C}} \times 21.7 ^{\circ}\text{C} = 344.4 \text{ m/s.}$$

$$\text{Percent error} = 0.03\%$$

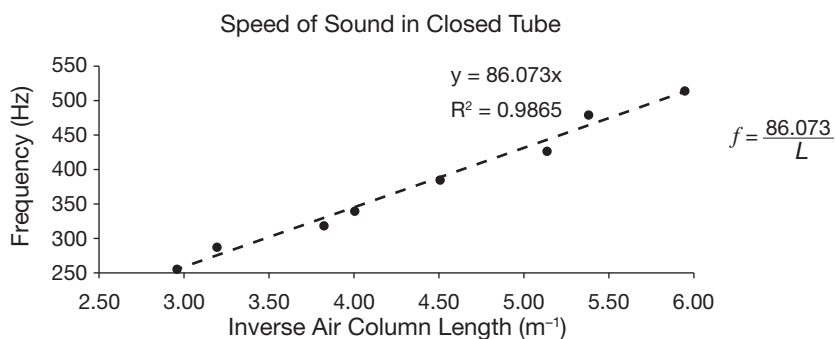
Systematic sources of error include the accuracy of each individual tuning fork. Relative humidity was also not factored in for the speed of sound. Random sources of error include the precision of the metric ruler and finding the exact length of the air column where the sound was loudest. The tuning fork needed to be struck repeatedly as the sound diminished over time. The water in the clear tube formed a meniscus, and the bottom of the meniscus changed somewhat depending on exactly how the PVC pipe was held—whether in the center of the clear tube or nearer to one side.

Teacher's Notes *continued*

Sample Data for Guided-Inquiry Activity

Temperature of air in tube: 21.7 °C.

Tuning Fork Frequency, f (Hz)	Air Column Length, L (m)			Calculated Wavelength, $\lambda = 4L$ (m)	Calculated Speed of Sound, $v = f\lambda$ (m/s)
	Trial 1	Trial 2	Average		
256	0.345	0.330	0.338	1.35	345.6
288	0.315	0.310	0.313	1.25	360.0
320	0.264	0.260	0.262	1.05	336.0
341.3	0.246	0.254	0.250	1.00	341.3
384	0.222	0.221	0.222	0.89	341.8
426.7	0.194	0.196	0.195	0.78	332.8
480	0.185	0.186	0.186	0.74	355.2
512	0.166	0.169	0.168	0.67	343.0



Answers to AP Physics 1 Review Questions

- Compare the accuracy of the two methods for determining the speed of sound in air—timing echoes and achieving resonance in a closed tube. How might the accuracy of each method be improved?

Using tuning forks to achieve resonance in a closed tube was more accurate than timing echoes, 0.03% error compared to 4%, respectively. The greatest source of error for the Introductory Activity was the timing of the claps. The claps are louder than the echoes, and the cadence was rather fast, so it was hard to tell if the cadence was exactly right. Using a digital microphone with a computer program to time an echo would provide more accurate results. The accuracy of the tuning fork method was excellent. It can be noted that calculating the speed of sound for each individual tuning fork resulted in a range of 332.8 – 360.0 m/s, with the greatest percent error of 4.5%, which is still very good. The accuracy of the tuning forks could be verified with an oscilloscope.

- How would the slope of graphed data for the guided-inquiry activity change if hot water were used instead of room temperature tap water?

If hot water had been used, then the temperature of the air in the PVC pipe would have been warmer. Therefore, the speed of sound would have been greater than at room temperature, and the slope of the graph would have been steeper.

- The PVC tube used in the guided-inquiry activity is 61 cm long. Of the eight tuning forks available in this activity, what is the lowest frequency tuning fork that would be able to resonate in the PVC tube at the next higher harmonic? Explain your reasoning.

The next higher harmonic for a closed tube is f_3 . Using the maximum length of the closed tube,

$$f_3 = \frac{3v}{4L} = \frac{3(344.4 \text{ m/s})}{4(0.61 \text{ m})} = \frac{1033.2 \text{ m/s}}{2.44 \text{ m}} = 423.4 \text{ Hz}$$

The lowest frequency tuning fork that could be used to achieve resonance at the third harmonic is 426.7 Hz. Any tuning forks with a lower frequency would require a pipe longer than 61 cm.

Teacher's Notes *continued*

4. A boat on Lake Michigan is outfitted with an echo sounder—a type of sonar unit that sends a wave pulse downward and then detects the pulse reflected from the lake bottom. If the time between the incident pulse and the detection of the reflected pulse is 0.27 seconds, how deep is the lake bottom, assuming the speed of sound in the water is 1500 m/s?

$$2d = 1500 \text{ m/s} \times 0.27 \text{ s}$$

$$d = 202.5 \text{ m}$$

5. Most orchestras tune to concert pitch, which is 440 Hz.

- a. A clarinet is considered a closed tube, since the musicians's mouth covers the vibrating reed on the mouthpiece. What is the length of the air column in the clarinet when it is in tune at concert pitch? Assume the temperature of the room is 20 °C.

$$f = v/4L$$

$$L = v/4f$$

$$L = \frac{343.4 \text{ m/s}}{4(440 \text{ Hz})}$$

$$L = 0.2 \text{ m}$$

- b. A flute is an open tube. What is the length of the air column in the flute when it is in tune at concert pitch in the same room?

$$f = v/2L$$

$$L = v/2f$$

$$L = \frac{343.4 \text{ m/s}}{2(440 \text{ Hz})}$$

$$L = 0.4 \text{ m}$$

- c. A piccolo is like a small flute, just 32 cm long. Can the piccolo tune to the fundamental frequency of concert pitch in the same room? If not, how can a piccolo tune with the orchestra?

$$f = \frac{343.4 \text{ m/s}}{2(0.32 \text{ m})} = 537 \text{ Hz}$$

No, the lowest fundamental frequency at which the piccolo can play is 537 Hz. Therefore, the piccolo would have to play at the second harmonic of concert pitch, or 880 Hz ($f_2 = 2f_1$).

Reference

AP Physics 1: Algebra-Based and Physics 2: Algebra-Based Curriculum Framework; The College Board: New York, NY, 2014.
HyperPhysics. Speed of Sound in Air. <http://hyperphysics.phy-astr.gsu.edu/hbase/sound/souspe.html> (accessed June 2014).

Speed of Sound—Advanced Inquiry Laboratory Kit and supporting supplies are available from Flinn Scientific, Inc.

Catalog No.	Description
AP7735	Speed of Sound—Advanced Inquiry Laboratory Kit
AP1037	Clamp, Universal Extension Clamp, 2"
AP8219	Clamp Holder
AP8226	Support Stand
AP6422	Tuning Fork Activator
AP9039	Rubber Mallet
AP4685	Ruler, Metric, 12"
AP6323	Tape Measure, 30 m

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