Hexaflexagon

Critical Thinking and Problem Solving

Introduction

Students use problem-solving strategies to build a mathematical puzzle called the hexaflexagon, a hexagon with two sides of different colors that can be "flexed" to produce six different sides of six different colors.

Concepts

- Critical thinking
- Momentary diversion

Materials

Adding machine paper tape roll
Crayons or markers, 6 different colors
Protractor
Scissors
Tape

Safety Precautions

Although this activity is considered nonhazardous, please follow all normal classroom or laboratory safety guidelines.

Procedure

1. Obtain a long piece from a roll of adding machine paper tape. Fold back one end of the tape to form a 60° angle as shown in Figure 1. It is not necessary for the angle to be exact, but the closer the approximation, the neater the final hexaflexagon will be and the easier it will be to "flex." Use a protractor if desired.

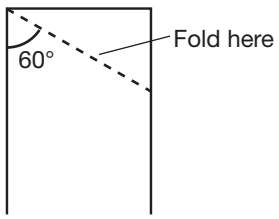

Figure 1.

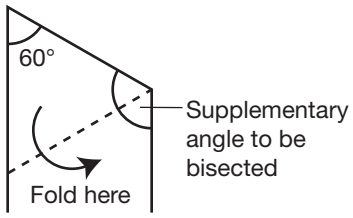

Figure 2.

2. Bisect the supplementary obtuse angle opposite the 60° angle just formed by folding the previously folded end up to meet the top edge of the paper (see Figure 2). By now, you should have folded two equilateral triangles on the paper, one on top of the other.
3. Continue bisecting the obtuse angles until you have folded 18 equilateral triangles on the paper. Open or unroll the tape and cut off the extra sides of the paper and discard them. Your tape roll should resemble Figure 3 (see step 4).
4. Number the triangles on the paper as shown in Figure 3. If desired, color all the triangles labeled "1" the same color, and so on. Alternatively, the numbered triangles can be decorated with specific patterns or details to distinguish them from one another.

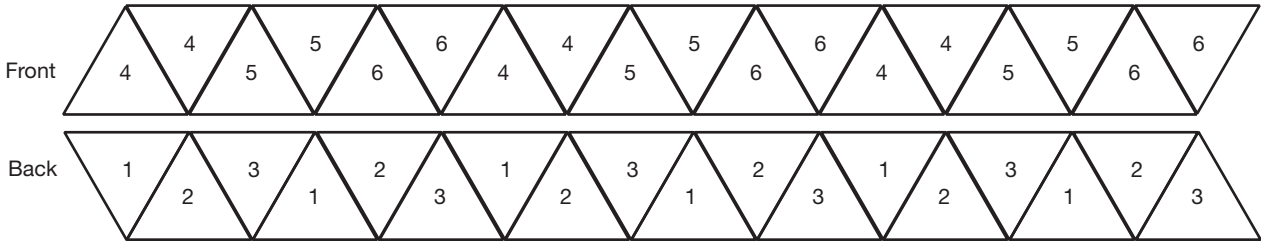

Figure 3.

Publication No. 91414
061616

5. To fold the hexaflexagon, begin by turning the paper so that the side numbered 4–6 is facing up. Fold the strip such that the first number "4" triangle touches the adjacent number "4" triangle face to face.
6. Repeat step 5 such that the number "5" triangle touches the adjacent number "5" triangle face to face. Flip the paper and continue with the following number "6" triangles.
7. Repeat steps 5–6, folding the like faces together until the strip of paper is half as long and twice as thick (see Figure 4 below).

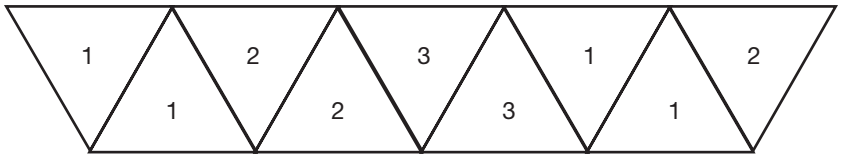

Figure 4.

8. Fold the first number "2" triangle such that it touches the adjacent number "2" triangle face to face. Your hexagon is beginning to take shape! (see Figure 5).
9. Turn the paper over and repeat step 8 with the following two number "2" triangles. You should now have a hexagon composed of six triangle faces. All but one of these faces will be numbered the same, as shown in Figure 6.
10. Tuck the differently-numbered triangle face under such that all of the numbered triangle faces are the same on each side of the hexaflexagon.
11. Tape the hexaflexagon together along the outside edges of the triangles, folding the tape over the edge from front to back (see Figure 7).
12. To "flex" the hexaflexagon, choose two adjacent triangles and push the outside edges down to form a roof-top ridge. Push the opposite vortex down and toward the center, then open the hexaflexagon from the middle like a flower as shown in Figure 8. You should now have a new face of the hexaflexagon facing up, with all triangle faces numbered the same, as before.

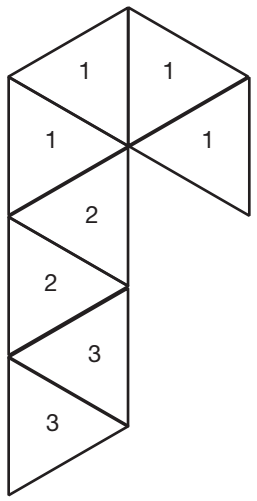

Figure 5.

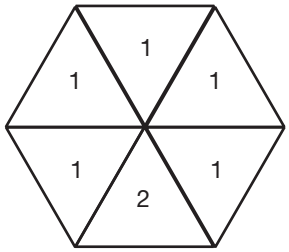

Figure 6.

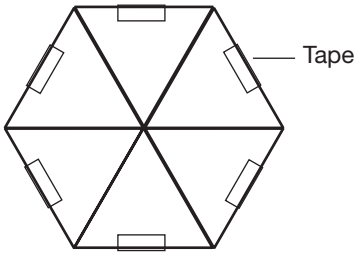

Figure 7.

13. Continue to "flex" the hexaflexagon to expose new numbered faces. The hexaflexagon will flex on all edges on some faces, but only on certain edges on other faces. If you reach an edge that will not flex, simply rotate the hexaflexagon and flex a different edge.

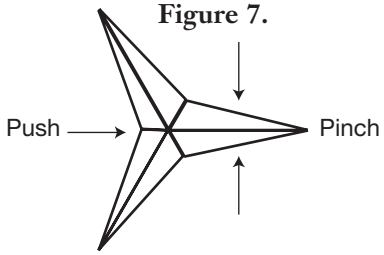

Figure 8.

Disposal

None required—save all materials for future use.

Tips

- Hexaflexagons may be purchased from magic shops (as Hexaflexagons or Flex-a-Colors) if you are unable to make your own. However, it may help students understand how the hexaflexagon works if they are able to see how it is constructed or even construct one themselves.
- Adding machine paper tape rolls are available from an office supply store.
- If the hexaflexagon was folded imperfectly, it may be difficult to "flex" and reveal all six faces. If this happens, use a hole-punch to cut out the outside corners of the hexaflexagon.

- All six colored or numbered faces can be revealed in sequence, although three of the faces will appear more often than the other three faces. To reveal this sequence, simply "flex" the hexaflexagon by pinching the same edge each time. If one edge does not flex, rotate the hexaflexagon and pinch an adjacent edge. Continue until all six faces are revealed in the fol lowing order:

  1–3–6–1–3–2–4–3–2–1–5–2–1

- This activity can also be used to teach acid-base chemistry. Fill six beakers with six different indicators to form six different colors when the same solution is poured into each beaker (see "The 'Rainbow Connection'" in *Acid–Base Indicators*, part of the Flinn Scientific eLearning Video Series). Use the hexaflexagon to show which colors will be formed by each solution.

Discussion

The hexaflexagon was first developed in the fall of 1989 by Arthur H. Stone, Bryant Tuckerman, and Richard P. Feynman, all graduate students at Princeton University, and the Princeton mathematics professor John W. Turkey. There are many different types of flexagons with differing numbers and types of faces; the type of flexagon used in this demonstration is called a "hexa-hexa-flexagon," due to the six possible hexagon faces (the six colors) formed by "flexing" the hexagon. There are also flexagons called "trihexaflexagons," which are hexaflexagons with only three possible faces. The complete mathematical theory behind the hexaflexagon was given by Turkey and Feynman, although students do not necessarily need to understand the theory in order to see the pattern of faces that emerges when the hexaflexagon is "flexed" a specific way each time. When the same edge or an adjacent edge is pinched each time the hexaflexagon is "flexed" and opened, the following sequence of faces emerges (see *Tips* section):

1–3–6–1–3–2–4–3–2–1–5–2–1

Slowly flex the hexaflexagon for your students and challenge them to predict which colors will show up each time. Students may catch the pattern after a few runs and may be able to successfully predict the repeating sequence of colors.

Besides helping students develop important critical thinking skills, the hexaflexagon can also be used as a "momentary diversion." This teaching technique was introduced by Hubert N. Alyea, a chemistry professor at Princeton University and the inspiration behind the title character in the Walt Disney film *The Absent-Minded Professor*. Alyea believed that all students, no matter the ability, learn best in small time increments of about 15–20 minutes. Thus, he concluded that it is essential to provide momentary breaks or lapses in the lesson plans to let students relax for a bit while still keeping them engaged and thinking. Alyea recommended using songs, anecdotal stories, or chemical demonstrations to use as temporary, yet educational breaks between the learning intervals. This hexaflexagon activity is a great way to do just that. Students will be using strategies from the scientific method as well as critical thinking skills without necessarily realizing they are doing so as they try to solve the mathematical puzzle.

Connecting to the National Standards

This laboratory activity relates to the following National Science Education Standards (1996):

Unifying Concepts and Processes: Grades K–12
Evidence, models, and explanation
Content Standards: Grades 9–12
Content Standard A: Science as Inquiry

References

Gardner, Martin. *Hexaflexagons and Other Mathematical Diversions.* University of Chicago Press, 1988.

Zimmerman, Joe. Hexa-hexa-flexagons Handout. Central Academ, Des Moines, IA.

Flinn Scientific—Teaching Chemistry™ eLearning Video Series

A video of the *Hexaflexagon* activity, presented by Jeff Hepburn, is available in *Critical Thinking and Problem Solving* and in *Momentary Diversions*, part of the Flinn Scientific—Teaching Chemistry eLearning Video Series.