How Tall Is That Tree?

Introduction

While on a hike you see a very tall pine tree ahead. Just how tall is the tree? Construct a handy instrument known as a clinometer and use it to indirectly measure the height of tall objects such as trees and buildings.

Concepts

- Indirect measurement
- Triangulation

Background

The height of an object such as a tree or building may be determined using a technique called *triangulation*. This method involves creating an imaginary *right triangle* (one angle of the triangle is 90°) in which the observer stands at a distance "*x*" from the point that lies directly underneath the object. If the object is a tree or building, *x* represents the distance between the observer and the base of the tree or building. The line of sight between the observer and the height of the object forms the elevation angle, θ (Greek lowercase *theta*), with respect to line *x*. The line of sight represents the side of the triangle opposite the right angle, which is known as the *hypotenuse*. With *x* representing the base of the triangle and the line of sight representing the hypotenuse, then the height of the object from the observer's eye, *h*, is the third side of the triangle (see Figure 1).
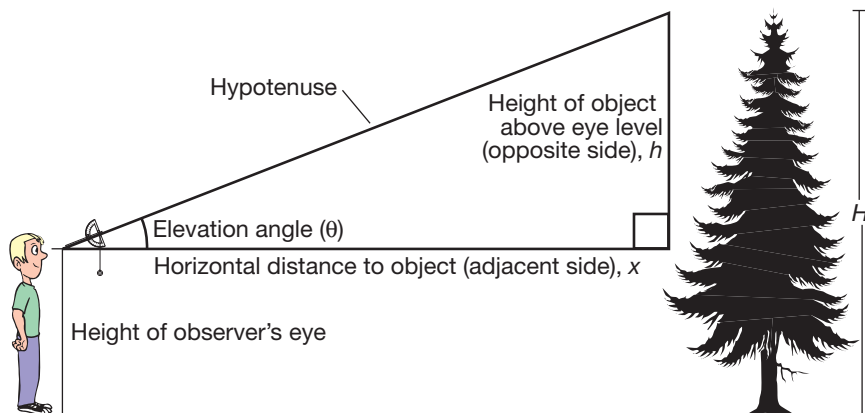

Figure 1.

To determine the height of an object, the distance from the observer to the object is first measured using a meter stick or tape measure. Next, angle θ is measured with a clinometer. The line of sight is determined by looking through a tube or site on the clinometer. In order to calculate the height of the object, the tangent of θ (tan θ) must be known. The tangent of an angle is a ratio determined by dividing the side opposite the angle (in this case the height of the object) by the side adjacent to the angle, distance *x* (Equation 1).

$$\text{Tan } \theta = \frac{\text{Side opposite angle } \theta}{\text{Side adjacent to angle } \theta} = \frac{\text{Height of object from eye level } (h)}{\text{Horizontal distance } (x)} \qquad \textit{Equation 1}$$

Since the length of the opposite side of the triangle is unknown, a scientific calculator with a tangent key may be used to find the value of tan θ. The height is then calculated using Equation 2.

$$h = x(\tan \theta) \qquad \textit{Equation 2}$$

With the clinometer held at eye level, the value of h is actually the height of the object from the height of the observer's eye, not from ground level (see Figure 1). Therefore, one more step is needed to determine the total height, *H*, of the object. The distance from the ground to the observer's eye must be added to *h* (Equation 3).

$$H = x(\tan \theta) + \text{distance from ground to eye} \qquad \textit{Equation 3}$$

Materials

Drinking straw	Scissors
Meter stick or tape measure	String, 30 cm
Protractor	Tape, cellophane
Scientific calculator	Washer, ¾″ diameter

Safety Precautions

The materials in this laboratory are considered nonhazardous. Use caution when sighting through the straw to avoid contact with the eye and do not look directly at the sun. Wear safety glasses. Please follow all laboratory safety guidelines.

Preparation

Make a clinometer

1. Obtain a 30-cm piece of string and a washer. Tie one end of the string to the washer, knotting securely.

2. Tie the free end of the string through the hole in the protractor or around the base at the halfway mark. Make sure the weighted string will hang on the front side of the protractor.

3. Position the knot of the string at the hole and tape the looped part of the string to the protractor. *Note:* When the protractor is held with the straight edge in a vertical position, the weighted string should hang down freely along the 0° line (see Figure 2).

4. Lay the straw across the back side of the protractor so the straw lines up with the 90° mark and the hole (see Figure 3).

5. Position the straw so that it extends approximately one centimeter beyond the straight edge of the protractor.

6. Use cellophane tape to secure the straw to the protractor as shown in Figure 3.
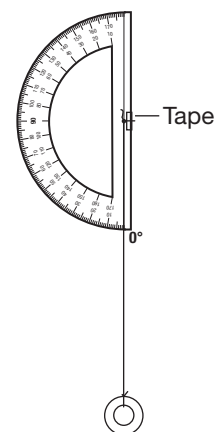
Figure 2.

Procedure

Determine the height of an object

1. Using a meter stick or tape measure, measure and record the height of the observer's eye from the ground in meters.

2. Measure the horizontal distance from the observer to the base of the object (ground level). Record the distance in meters—this value represents the adjacent side of the triangle.

3. The observer holds the clinometer so the weighted string hangs freely as in Figure 2. The straw should be parallel to the ground. *Note:* Make sure the observer is standing at the distance measured in step 2.

4. Holding the protractor, the observer looks through the long end of the straw (the sight) and tilts the clinometer so the top of the object is visible through the sight (see Figure 1). *Caution:* Hold the end of the straw away from the eye while sighting the object.

5. Once the top of the object has been sighted, a partner notes the degree mark through which the string passes. *Note:* This angle will be between 0° and 90° and is referred to as angle theta (θ). Record the angle.

6. Using a scientific calculator with a tangent key, find the tangent to angle θ for the object. Record the tan θ in the data table.

7. Use Equation 1 from the *Background* section to determine the length of the opposite side for the object. Record the length of the opposite side in the data table.

8. Add the height of the observer's eye to the length of the opposite side (Equation 2) to determine the total height of each object. Record the total height for each object in the data table.
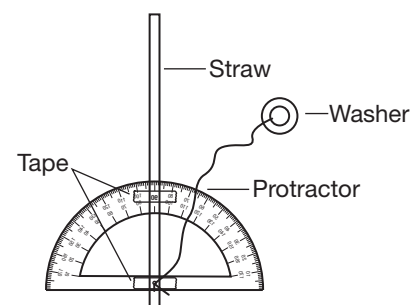
Figure 3.

Sample Data Table *(Student data will vary.)*

Object	Height of Observer's Eye (m)	Adjacent Side, x (m)	Angle θ	Tan θ	Opposite Side, h (m)	Total Height, H (m)
Clock on wall	*1.57*	*3.05*	*9.0°*	*0.1584*	*0.483*	*2.05*
Flagpole	*1.46*	*11.6*	*27.0°*	*0.5095*	*5.91*	*7.37*

NGSS Alignment

This laboratory activity relates to the following Next Generation Science Standards (2013):

Disciplinary Core Ideas: Middle School
MS-PS2 Motion and Stability: Forces and Interactions
  PS2.A: Forces and Motion
MS-LS1 From Molecules to Organisms:
      Structures and Processes
  LS1.A: Structure and Function
  LS1.B: Growth and Development of Organisms

Disciplinary Core Ideas: High School
HS-PS2 Motion and Stability: Forces and Interactions
  PS2.A: Forces and Motion
HS-LS1 From Molecules to Organisms: Structures and Processes
  LS1.A: Structure and Function
  LS1.B: Growth and Development of Organisms

Science and Engineering Practices
Analyzing and interpreting data
Using mathematics and computational thinking

Crosscutting Concepts
Patterns

Tips

- Student-made clinometers are useful for practicing measurement skills and finding the height of tall objects. They may also be used to determine the maximum altitude of a launched model rocket.
- Be sure students know how to use the tangent key on their scientific calculators. If calculators are not available, most computers have a calculator program with a scientific view. Simply type in the numeral for the angle and click on the tangent key. Tangent tables for whole-number angles from 0 to 90 degrees may be found online. See http://math.com/tables/trig/tables.htm for one example (accessed July 2015).
- For practice indoors, tape a picture of a tree or other object near the ceiling of the classroom. If the school gym is available, students may also determine the height of the basketball rim or the top of the backboard.
- Use this activity to emphasize that it is not always possible to obtain direct observations and measurements of many objects, such as very small, very large, or faraway objects. Indirect measurements are frequently used in science to learn more about these types of objects. Challenge students to give examples!
- This activity is available from Flinn Scientific as a student laboratory kit, AP7386, *Make Your Own Clinometer*. Kit includes a reproducible student worksheet and enough materials for 30 students working in pairs.

References

Bilash, B.; Maiullo, D. *A Demo a Day—A Year of Physics Demonstrations;* Flinn Scientific: Batavia, IL, 2009; p 2.

Materials for *How Tall Is That Tree?* are available from Flinn Scientific, Inc.

Catalog No.	Description
AP7386	Make Your Own Clinometer—Student Laboratory Kit
AP9286	Protractor
AP6025	Straws, Wrapped, 500/pkg
AP5402	Meter Stick, Hardwood
AP6323	Tape Measure, Wind-Up Type, 30 m
AP4823	String, ball

Consult your *Flinn Scientific Catalog/Reference Manual* for current prices.