PSworks Parabola

Demonstration Kit

Introduction

What shape is the path that a projectile follows? Is the speed of a projectile at the beginning of its journey really the same as at the end? These concepts are often difficult to comprehend without the help of a visual aid. Use the PSworks Parabola and Photogate Timer to safely and effectively show these important principles of projectile motion.

Concepts

- Projectile motion
- Parabolic motion
- Newton's laws of motion
- Conservation of energy principle

Background

Parabolic motion

All objects fall toward the Earth at the same increasing rate (in a vacuum). That is, all objects will accelerate toward the Earth equally, regardless of their mass, as a result of the force due to gravity. In a vacuum, where there is no drag friction due to air, a heavy hammer will fall at exactly the same rate as a light feather. At the surface of the Earth, the acceleration toward the center of the Earth experienced by all objects is measured to be (on average) 9.8 m/s^2 (32 ft/s^2).

A force is considered a vector quantity, meaning it has a magnitude and a direction. Therefore, a force can be separated into horizontal and vertical components that are independent of each other. Thus, for a force that pushes a ball up at an angle with respect to the ground, the force is said to have one force component that is vertical and one that is horizontal. Both force components depend on the total force and on the angle of the force with respect to the ground, but they are independent of each other. A vertical force will have no effect on the magnitude of the horizontal force component, and vice versa.

Projectiles are the simplest objects to use to demonstrate the principle of horizontal and vertical force independence. When an object is in freefall, the only force that acts on the object is the force due to gravity pulling it downward. If the object also has horizontal movement, this horizontal movement will not be affected by the downward force due to gravity. The equations of motion of a projectile are given by Equations 1 and 2.

$$y = \tfrac{1}{2}gt^2 + v_{yo}t + y_o \qquad \textit{Equation 1}$$

$$x = v_{xo}t + x_o \qquad \textit{Equation 2}$$

In Equations 1 and 2, y represents the vertical position, g is the acceleration due to gravity, t is the time, v_{yo} is the initial vertical velocity, y_o is the initial vertical position, x is the horizontal position, v_{xo} is the initial horizontal velocity and x_o is the initial horizontal position. Notice that Equation 2 (the horizontal, or x-direction, equation) does not contain an acceleration component, meaning the speed of the projectile in the horizontal direction remains constant. Solving Equation 2 for t and substituting into Equation 1 yields:

$$y = \tfrac{1}{2}g[(x - x_o)/v_{xo}]^2 + v_{yo}[(x - x_o)/v_{xo}] + y_o \qquad \textit{Equation 3}$$

Equation 3 represents a quadratic equation that defines a parabolic curve. So, the path that a projectile will follow is that of a parabola. In this activity, when the ball bearing is positioned at the proper location for the force applied by the launching mechanism, the ball bearing will become a projectile that clearly follows the parabolic path of the wood track.

The general equation of motion for a falling object is given by Equation 4, where v_f represents the final velocity, v_i is the initial vertical velocity, g is the acceleration due to gravity, d_y is the distance the object travels (falls).

$$v_f^2 - v_i^2 = 2gd_y \qquad \textit{Equation 4}$$

Equation 4 can be used to calculate the expected final velocity of an object that falls a certain distance (height). And, if an object is launched to reach the same height then the object must be launched with an initial speed that is equal to the speed it had at the end of its fall. As an example, if an object were to fall from 10 m with an initial vertical velocity of 0 m/s, then the object's velocity after 10 m will be equal to 14 m/s. In order to launch the object to a height of exactly 10 m, then the object will need an initial vertical velocity of 14 m/s. This is also in agreement with the conservation of energy principle.

Conservation of energy

The PSWORKS Parabola can also be used to demonstrate the conservation of energy principle. The law of conservation of energy states that energy cannot be created or destroyed—only converted between one form and another. In order to raise a ball to the release point at the top of an inclined plane, one must exert energy. The energy used to raise the ball becomes "stored" with the ball. This stored energy is referred to as gravitational *potential energy* (*PE*). The potential energy of the ball is related to its height and weight. In general, potential energy is equal to the weight of an object, which equals the mass (m) times the acceleration due to gravity (g), multiplied by the relative height (h) of the object ($PE = mgh$).

As the ball moves down the curved track, its potential energy is converted into *kinetic energy* (energy of motion). For the rolling ball, the initial potential energy has been converted to both rotational and linear kinetic energy, both of which are proportional to one-half the square of the rotational and linear speed, respectively.

In this demonstration, the ball will roll down a track without slipping. The point on the ball in contact with the surface of the track will be instantaneously at rest with respect to the track. The frictional force between the surface of the rolling object and the surface of the track acts against, and balances, the force due to gravity pulling the object down. Since no slipping occurs between the two surfaces, energy will not be dissipated or lost as heat and (theoretically) all the potential energy the ball has at the top of the track will be converted into kinetic energy at the bottom. Therefore, the ball will have the same energy at the bottom of the track that it had at the top of the track. This energy will be converted back into potential energy as the ball begins to move up the track. The ball will not be able to rise any higher than the initial height because the maximum potential energy at the top of the incline will be the same as the initial potential energy before the ball begins to roll down the curve.

Materials (for each demonstration)

- Cup, plastic, or towel (to "catch" the ball)
- Knob with threaded insert*
- Level (optional)
- Neodymium magnet*
- PSWORKS Parabola*
- PSWORKS Photogate Timer
- PSWORKS Support Stand
- Scissors
- Steel ball bearings, ½″ dia., 4*
- Tape, electrical*

**Materials included in kit.*

Safety Precautions

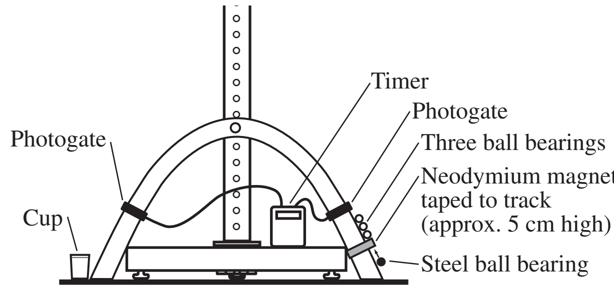
The materials in this demonstration are considered safe. The ball bearing projectile can bounce in random directions. Wear safety glasses during the demonstration and students who are sitting near the demonstration should also wear safety glasses.

Demonstration 1. Parabolic Motion

Preparation

1. Connect the Parabola to the PSWORKS Support Stand using the knob with threaded insert. Do not tighten the knob completely. Leave the Parabola with enough "play" to adjust the position of its legs and its vertical orientation. Make sure the legs of the Parabola are flat on the tabletop and that the Parabola stands upright. Use a level, if necessary, to adjust the Parabola's orientation appropriately.
2. Secure the photogates to the Parabola at the two 10-cm marks (See Figure 1 on page 3). Make sure the open slits in the photogates line up with the markings on the Parabola.
3. Connect the photogates to the timer.
4. Obtain the neodymium magnet and electrical tape.

5. At approximately the 5-cm mark on the PSWORKS Parabola, use one piece of the electrical tape to secure the neodymium magnet to the track. See Figure 1. It may be necessary to cut the electrical tape in half along its center line so the tape does not extend over the ends of the magnet.
7. Place a plastic cup or towel at the other end of the track to “catch” the ball bearing.
6. Place three ball bearings on the end of the neodymium magnet facing up the track (see Figure 1).
8. Holding the steel ball bearing at the bottom of the Parabola track, use one finger to slowly raise the ball bearing along the track towards the neodymium magnet.



Photogate slots must be at the same height.

Figure 1.

9. Once the ball bearing is close enough to the magnet, the ball will be quickly pulled toward the magnet and the resulting collision will shoot the last ball bearing in the series up the Parabola. If necessary, adjust the position of the neodymium magnet until the ball bearing skims across the surface of the track, makes it to the top of the Parabola and then falls to the other side. This may require a bit of trial and error, but the position of the magnet should be within 2–3 cm of the initial 5-cm position. To obtain the proper flight path, it is sometimes easier to give the ball bearing a slight push into the magnet. The extra force provides more control over how much energy is used to launch the ball bearing.
10. Once the neodymium magnet is in the proper position, use a few more pieces of electrical tape to secure it firmly to the Parabola.

Procedure

1. Turn on the timer and select GATE mode.
2. Place the three ball bearings on the neodymium magnet.
3. Holding the steel ball bearing at the bottom of the Parabola track, raise the ball bearing towards the neodymium magnet using one finger in the same manner that was used during the *Preparation*.
4. Watch the ball shoot up the track and follow the path of the parabola.
5. Record the “gate times” for both photogates on the board. How do the two times compare? *(The time measurements should be nearly equal, ±0.001 s.)*
6. Reset the demonstration and repeat as often as necessary (remove the steel ball bearing from the magnet and reset the Photogate Timer to GATE mode). Record the “gate times” for both photogates on the board again, as desired.
7. Place the ball bearing at the top of the parabola and allow it to roll down one side.
8. Record the “gate time” for the rolling ball bearing starting from the top of the parabola. How close is it to the launched “gate times?” *(The time measurements should be relatively close to the other time measurements.)*
9. Repeat steps 3–8 as often as necessary and discuss the results with students.

Demonstration 2. Conservation of Energy

1. Securely connect the PSWORKS Parabola, legs pointing up, to the PSWORKS Support Stand (see Figure 2). Angle the Parabola so that the legs are not uniform in height.
2. Position the ball on the steeper side of the track, just below the height of the other leg. Ask students to predict if the ball will roll off the track.
3. Release the ball and note the height the ball reaches on the other side. Did the ball leave the track?
4. Discuss the concept of conservation of energy with your students.

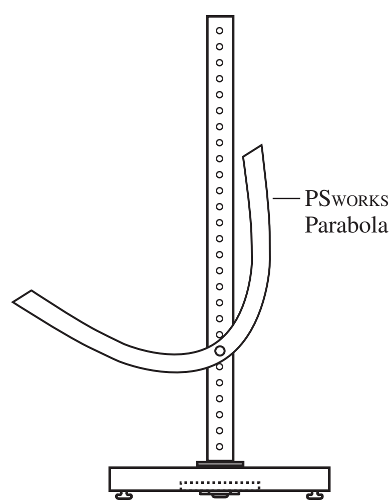


Figure 2.

Disposal

The materials should be saved and stored in their original container for future use.

Tips

- This kit contains enough materials to perform the demonstrations indefinitely. This demonstration may still be performed without the Photogate Timer. Position the magnet higher along the track so that the ball nearly shoots off the track so students can observe the parabolic shape better.
- It may be best to position the magnet lower on the track. Then, in order to launch the ball bearing with enough speed, push the ball into the magnet instead of just allowing it to be pulled up. This usually provides better control and more consistent results.
- The time measurements by the Photogate Timer will be affected by how much of the ball bearing crosses the light sensor. Make sure the center diameter of the ball bearing crosses the slots of both photogates. If one photogate measures a "gate time" that varies significantly from the other, then most likely the photogates are not "seeing" the same diameter ball bearing. In other words, the full diameter of the ball bearing may cross the first photogate but only the bottom portion of the ball bearing, and thus a smaller diameter, crosses the second photogate resulting in a much quicker time measurement (and apparent faster ball bearing). Adjust the photogates until both "see" the same ball diameter.
- Keep neodymium magnets away from computer disks or other magnetic strips such as credit cards. They will quickly erase the magnetized data.
- The launching mechanism may appear to break the laws of physics, especially the laws of conservation of energy and momentum, because the ball bearing appears to gain free energy after the collision. However, no laws have been violated. Magnetic fields store potential energy the same way that a spring does when stretched or compressed, or that a ball does when it is held above the ground. When the ferromagnetic metal ball is far away from the magnet (at "infinity"), the ball has a small amount of magnetic potential energy. As the steel ball bearing is pulled toward the magnet, the speed of the ball bearing increases significantly, and often unnoticed, over the short distance it travels. The ball then hits the magnet and comes to a stop. Since energy and momentum must be conserved in an elastic collision, the energy transfers through the magnet to the last ball bearing. And since the last ball bearing is separated from the magnet, it is not affected by the magnetic field as strongly and it shoots off at great speed.
- Mathematically, a general conic section is defined by a real polynomial (Equation 5) with a discriminant (*D*) equal to Equation 6.

  $$ax^2 + bxy + cy^2 + dx + ey + f = 0 \qquad \textit{Equation 5}$$

  $$D = b^2 - 4ac \qquad \textit{Equation 6}$$

  If *D* is equal to zero, then the conic section polynomial is defined by a parabola. Substituting the values in Equation 3 for the constants in Equation 5, *a* equals $\frac{1}{2}g/v_{xo}^2$, and *b* and *c* both equal zero. Therefore, the discriminant of Equation 3 is equal to zero, proving that Equation 3 defines a parabolic shape.

Connecting to the National Standards

This laboratory activity relates to the following National Science Education Standards (1996):

Unifying Concepts and Processes: Grades K–12
Systems, order, and organization
Evidence, models, and explanation
Constancy, change, and measurement
Content Standards: Grades 5–8
Content Standard B: Physical Science, understanding of motions and forces, transfer of energy
Content Standard E: Science and Technology
Content Standards: Grades 9–12
Content Standard B: Physical Science, motions and forces, conservation of energy and increase in disorder
Content Standard E: Science and Technology

Answers to Worksheet Questions *(Student answers will vary.)*

Post-Demonstration Questions

1. Describe the mathematical or geometric shape that the projectile ball followed.

   The projectile ball bearing follows the same path as the wood track, which happens to be a parabolic shape.

2. How did the speed of the ball bearing after it was launched compare to the speed it had as it was falling down the curved path? Explain in terms of the conservation of energy principle.

   The speed of the ball bearing is about the same at both ends of the path. The launch speed is about the same as the falling speed, when the timers are positioned at the same height. The conservation of energy principle states that energy cannot be created or destroyed, only converted between the various kinds of energy. The ball bearing is given a certain amount of kinetic energy after the collision, which determines the speed of the ball. The ball follows a uniform parabolic path so after it reaches the peak (and has zero vertical speed for an instant) the ball will fall following the same path. When the ball reaches the same it had initially, it will have the same speed because it has the same amount of kinetic energy.

3. Did the ball bearing roll off the end of the track during the conservation of energy demonstration? Explain in terms of the conservation of energy principle.

   No, the ball bearing did not roll off the end of the track, even though the other side had a shallower angle. Due to the conservation of energy principle, the ball released down the track will have energy equal to its initial potential energy based on the height above a reference point (without energy added due to a push). As the ball travels up the track it will rise to a height that gives the ball the same initial potential energy. This means the ball cannot rise higher than the initial height, no matter what path the ball takes (i.e., a shallower incline versus a steeper incline).

Materials for the *PSworks Parabola* are available from Flinn Scientific, Inc.

Catalog No.	Description
AP7208	PSworks Parabola
AP6998	PSworks Photogate Timer
AP6999	PSworks Support Stand

Consult your *Flinn Scientific Catalog/Reference Manual* for current prices.

Name: ______________________________

PSworks Parabola Worksheet

Post-Demonstration Questions

1. Describe the mathematical or geometric shape that the projectile ball followed.

2. How did the speed of the ball bearing after it was launched compare to the speed it had as it was falling down the curved path? Explain in terms of the conservation of energy principle.

3. Did the ball bearing roll off the end of the track during the conservation of energy demonstration? Explain in terms of the conservation of energy principle.